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Numerical Approximation Of Iterative Induced Sequence On Cone Metric Spaces

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Abstract
A cone metric space generalizes the conventional notion of a metric space by introducing distance measure within a real Banach space, E where a subset $P \subset E$ satisfying the properties of a cone governs the structure of the metric. In the present study, the cone metric is formulated such that the cone $P \subset \mathbb{R}^2$ comprises vectors with non-negative components. A rigorous verification established that the space $(\mathbb{R}^2, d)$ satisfies the fundamental axioms of a cone metric, namely positivity, symmetry, and the triangle inequality. Furthermore, the study investigates the numerical approximation of the convergence of an iterative sequence generated by a generalized Lipschitzian mapping, the simulation results demonstrate that the sequence $\{x_n\}_{n\geq 0}$ converges rapidly to a unique fixed point of the corresponding contraction mapping. This finding substantiates the theoretical construction of fixed point results in cone metric spaces and highlights their applicability in analyzing the stability and convergence behaviours of iterative processes.
Keywords: *Cone metric spaces, Fixed point theorem, Generalized Lipschitzian condition, Iterative numerical approximation*

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I. Introduction And Preliminaries

Let E always be a real Banach space and P a subset of E, then following Okeke (2019) [1] and Ilic, Rakocevic (2009) [2], Rezapour and Hamlbarani (2008) [3], P is called a cone if and only if:
(i) P is closed, nonempty, and $P \neq \{0\}$
(ii) $\{0,1\} \subset P$
(iii) $a, b \in \mathbb{R}, a, b \geq 0, x, y \in P \Rightarrow ax + by \in P$
(iv) $x \in P, -x \in P \Rightarrow x = 0$.

Several studies have employed analytical methods alongside software tools such as SPSS and MATLAB for the analysis and interpretation of modelling results. For example, Okeke (2020) [4], Okeke and Akpan (2019) [5], and Okeke and Ifeoma (2024; 2023) [6], [7] examined various modelling approaches applied to physical phenomena and the sensitivity of coronavirus disparities in Nigeria. Okeke et al. (2019) [8] utilized analytical theorems, including the fixed point theorem.

Okeke and Peters (2019) [9] emphasized numerical stability in physical flow applications using Software-assisted analysis tool. The principle of maximum was analytically applied to establish the uniqueness of solutions in metric spaces involving second-order linear Volterra integral equations.

Moreover, Okeke and Nwokolo (2025) [10] applied modelling techniques to healthcare patient scenarios, while Okeke et al. (2019) [11] used MATLAB for modelling and analyzing the dynamics of HIV infection. Okeke (2025) [12] used Quality Management (QM) software to solve innovative mathematical application of game theory to healthcare allocation problem.

Huang and Zhang (2007) [13] introduced cone metric spaces as an extension of conventional metric spaces. In a cone metric space X, the distance $d(x, y)$ between two elements x and y is defined as a vector in an ordered Banach space E. A mapping $T: X \to X$ is called contractive if there exists a constant $k \in [0,1)$ such that $d(Tx, Ty) \leq kd(x, y) \; \forall \; x, y \in X$ (Khojasteh et al., 2015) [14].

Following Chidume (2003) [15], let X be a nonempty set. Suppose the mapping $d: X \times X \rightarrow E$ E satisfies:
(i). $d(x,y) > 0$ for all $x, y \in X$, and $d(x,y) = 0$ if and only if $x = y$,
(ii). $d(x,y) = d(y,x)$ for all $, y \in X$,
(iii). $d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X, and the pair (X, d) is called a cone metric space. It is obvious that cone metric spaces generalize metric spaces.

In particular, let $X = \mathbb{R}$ and define a cone metric $d: X \times X \rightarrow \mathbb{R}^2 \subseteq$ E as:

$$d(x,y) = |x - y|.(1,1) \tag{1}$$

with the cone defined by: $P = \{(a,b) \in \mathbb{R}^2 : a \geq 0, b \geq 0\}$

II. Verification That ($\mathbb{R}$, d) Is A Cone Metric Space

Let $X = \mathbb{R}$, and define a distance function

$$d: X \times X \rightarrow \mathbb{R}^2 \text{ by: } d(x,y) = |x - y|.(1,1) = (|x - y|, |x - y|) \tag{2}$$

Let the cone $P \subset \mathbb{R}^2$ be defined as: $P = \{(a,b) \in \mathbb{R}^2 : a \geq 0, b \geq 0\}$, which induces a partial order on $\mathbb{R}^2$ given by: $(u_1, u_2) \leq (v_1, v_2) \Leftrightarrow v_1 - u_1 \geq 0 \text{ and } v_2 - u_2 \geq 0.$

We now verify the three axioms of a cone metric.

(i). Positivity

We have that: $d(x,y) = (|x - y|, |x - y|)$. Since $|x - y| \geq 0$, it follows that $d(x,y) \in$ P. Furthermore,
$d(x,y) = (0,0) \Leftrightarrow |x - y| = 0 \Leftrightarrow x = y$. Thus, positivity is satisfied.

(ii). Symmetry

We observe that: $d(x,y) = |x - y|(1,1) = |y - x|(1,1) = d(y,x)$. Hence, symmetry is satisfied.

(iii). Triangle Inequality

We need to verify that: $d(x,z) \leq d(x,y) + d(y,z)$.

Computing: $d(x,z) = |x - z|(1,1)$, $d(x,y) + d(y,z) = (|x - y| + |y - z|)(1,1)$.

By the triangle inequality in $\mathbb{R}$: $|x - z| \leq |x - y| + |y - z|$.

Multiplying both sides by $(1,1) \in P$, we get: $|x - z|(1,1) \leq (|x - y| + |y - z|)(1,1)$.

Thus, $d(x,z) \leq d(x,y) + d(y,z)$.

Triangle inequality holds with respect to the partial ordering defined by P.

Since all the three properties of a cone metric are satisfied, $(X = \mathbb{R}, d)$, with:
$d(x,y) = |x - y|(1,1)$ and $P = \{(a,b) \in \mathbb{R}^2 : a \geq 0, b \geq 0\}$ is a cone metric space.

Next, choosing the map:

$$T(x) = \frac{1}{2}x + 1 \tag{3}$$

which is a contraction with Lipschitz constant $k = \frac{1}{2} < 1$.

Next, we now show that the map $T(x) = \frac{1}{2}x + 1$ is a contraction with Lipschitz constant $k = \frac{1}{2}$, we need to demonstrate that there exists a constant $k \in (0,1)$ such that for all x, y in the domain, we have: $|T(x) - T(y)| \leq k|x - y|$

But:

$$|T(x) - T(y)| = |\frac{1}{2}x + 1 - \frac{1}{2}y - 1|$$

$$= |\frac{1}{2}x - \frac{1}{2}y| \tag{4}$$

$$= \frac{1}{2}|x - y|$$

Now, comparing $|T(x) - T(y)| \leq k|x - y|$ and equation (3), then $k = \frac{1}{2}$.

III. Methodology

This section explores an iterative process. Defining the iterative sequence $\{x_n\}$ by:

$$x_{n+1} = T(x_n)$$

Starting with:

$$x_0 = 0$$

Then from equation (3),

$x_1 = T(x_0) = \frac{1}{2} . 0 + 1 = 1$

$x_2 = T(x_1) = \frac{1}{2} . 1 + 1 = \frac{3}{2}$

$x_3 = T(x_2) = \frac{1}{2} . \frac{3}{2} + 1 = \frac{7}{4}$

$x_4 = T(x_3) = \frac{1}{2} . \frac{7}{4} + 1 = \frac{15}{8}$

$$\vdots$$

The fixed point is found by solving: $x^* = T(x^*) \Rightarrow x^* = \frac{1}{2}x^* + 1 \Rightarrow x^* = 2$

The mapping $T(x) = \frac{1}{2}x + 1$ is a contraction on $\mathbb{R}$, and thus the iterative sequence $x_{n+1} = T(x_n)$ converges to the unique fixed point $x^* = 2$ for any initial point $x_0 \in \mathbb{R}$.

Steps for Derivation of a Contraction

Step 1: Stating the definition of a contraction map

Let the mapping $T : \mathbb{R} \rightarrow \mathbb{R}$ be defined as: $T(x) = \frac{3}{10}x + \frac{7}{5}$

Let the mapping T be called a contraction if there exists a constant $k \in (0,1)$ such that: $|T(x) - T(y)| \leq k\,|x - y| \leq \forall\, x, y \in \mathbb{R}$.

Step 2: Computing the Difference: $|T(x) - T(y)|$

But:

$T(x) = \frac{3}{10}x + \frac{7}{5}$

$T(y) = \frac{3}{10}y + \frac{7}{5}$

$\Rightarrow |T(x) - T(y)| = |\left(\frac{3}{10}x + \frac{7}{5}\right) - \left(\frac{3}{10}y + \frac{7}{5}\right)|$

$|T(x) - T(y)| = |\frac{3}{10}x - \frac{3}{10}y| = \frac{3}{10}|x - y|$

Step 3: Comparing with the Contraction Condition

From the computation above, we have:

$|T(x) - T(y)| = \frac{3}{10}|x - y|$

This matches the contraction definition with: $k = \frac{3}{10}$

The contraction constant k for the mapping $T(x) = \frac{3}{10}x + \frac{7}{5}$ is: $k = \frac{3}{10}$

The fixed point is found by solving: $x^* = T(x^*) \Rightarrow x^* = \frac{3}{10}x^* + \frac{7}{5} \Rightarrow x^* = 2$

The mapping $T(x) = \frac{3}{10}x + \frac{7}{5}$ is a contraction on $\mathbb{R}$, and thus the iterative sequence $x_{n+1} = T(x_n)$ converges to the unique fixed point $x^* = 2$ for any initial point $x_0 \in \mathbb{R}$.

IV. Results

This section gives the metric space with Euclidean metric. Suppose, we let (X, d) be a complete metric space with the standard Euclidean metric $d(x, y) = |x - y|$, and let the function $T: X \rightarrow X$ be defined as: $T(x) = kx + c$ where $0 \leq k < 1$, then T is a generalized Lipschitzian map.

The work done by Okeke et al. (2020) [16] employed numerical simulation to determine the concentration of the material in relation to the independent variables. Numerical simulation was used to investigate the effect of temperature on concentration and to calculate the material's thermal diffusivity. Their results were presented through various plots at different temperature levels.

Hence, in the simulation for $(x) = kx + c$, let $k = 0.6$, $c = 2$ with $x_0 = 10$ and the sequence $\{x_n\}$ be defined by: $x_{n+1} = T(x_n) = kx_n + c$. The fixed point of T satisfies $x^* = T(x^*) \Rightarrow x^* = \frac{c}{1-k} = \frac{2}{0.4} = 5$. The computation of the sequence x_n for $T(x) = 0.6x + 2$.

We are given the recursive sequence:
$x_{n+1} = T(x_n) = 0.6x_n + 2$, with the initial condition: x_0= 10.

The step-by-step computations are as follows:
x_0= 10,
$x_1 = 0.6 \cdot 10 + 2 = 6 + 2 = 8$,
$x_2 = 0.6 \cdot 8 + 2 = 4.8 + 2 = 6.8$,
$x_3 = 0.6 \cdot 6.8 + 2 = 4.08 + 2 = 6.08$,
$x_4 = 0.6 \cdot 6.08 + 2 = 3.648 + 2 = 5.648$,
$x_5 = 0.6 \cdot 5.648 + 2 = 3.3888 + 2 = 5.3888$,
$x_6 = 0.6 \cdot 5.3888 + 2 = 3.23328 + 2 = 5.23328$,
$x_7 = 0.6 \cdot 5.23328 + 2 = 3.139968 + 2 = 5.139968$,
$x_8 = 0.6 \cdot 5.139968 + 2 = 3.0839808 + 2 = 5.0839808$,
$x_9 = 0.6 \cdot 5.0839808 + 2 = 3.05038848 + 2 = 5.05038848$
$x_{10} = 0.6 \cdot 5.05038848 + 2 = 3.03023309 + 2 = 5.03023309$

Table 1: Values of x_n for successive n under a generalized Lipschitzian map defined on the given theoretical bound

n	0	1	2	3	4	5	6	7	8	9	10
x_n	10.00000 000	8.00000 000	6.80000 000	6.08000 000	5.64800 000	5.38880 000	5.23328 000	5.13996 800	5.08398 080	5.05038 848	5.03023 309

For the theoretical Bound: $d(x_n, x^*) \leq \frac{k^n}{1-k} d(x_0, x_1)$
In this case:
$d(x_0, x_1) = |10 - 8| = 2$ and $\frac{1}{1-k} = \frac{1}{0.4} = 2.5$

$\Rightarrow d(x_n, x^*) \leq 2.5 \cdot k^n \cdot 2 = 5k^n$

Numerical Simulations

The convergence behaviour of the three different iterative sequences on cone metric spaces with the iteration of Lipschitzian Mapping are shown in the Figure 1 to Figure 3 below. The Figure 4 visualizes the metric cone P ⊂ $\mathbb{R}^2$ while Figure 5 shows the convergence of the sequence x_n to the fixed point x^*generated by a generalized Lipschitzian map for the function $T(x)$.

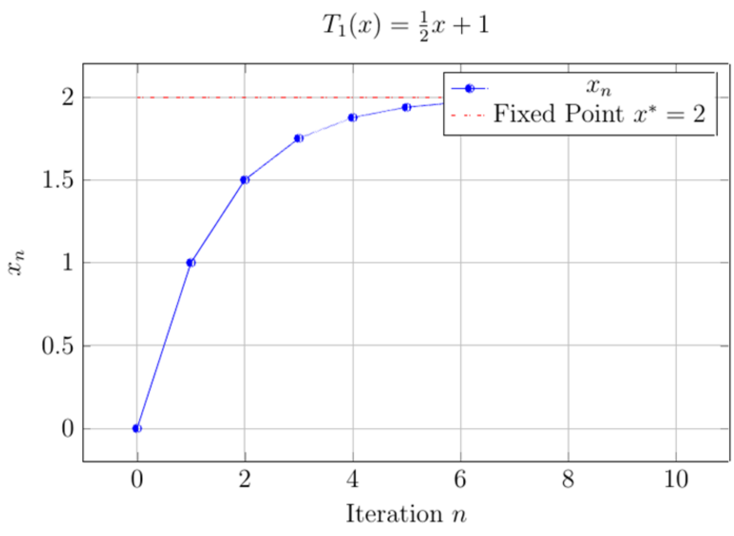

Figure 1: Plot of the iterative sequence x_n for $T_1(x) = \frac{1}{2}x + 1$ with fixed point $x^* = 2$.

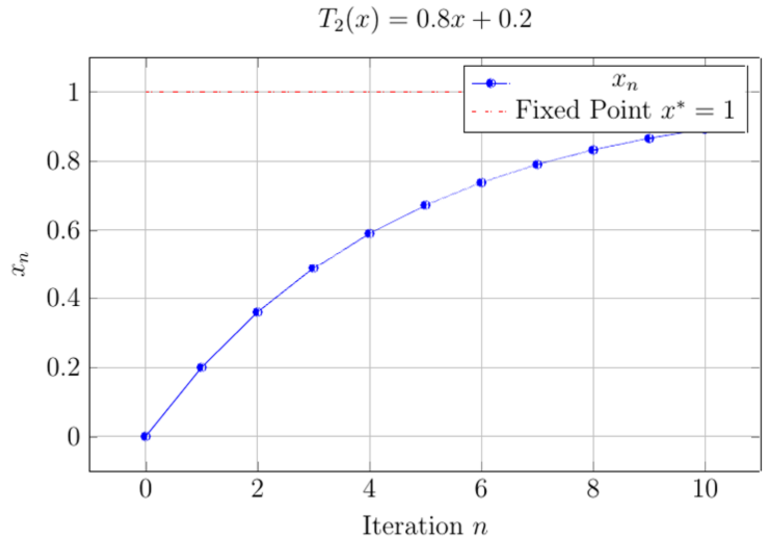

Figure 2: Plot of the iterative sequence x_n for $T_2(x) = 0.8x + 0.2$ with fixed point $x^* = 1$.

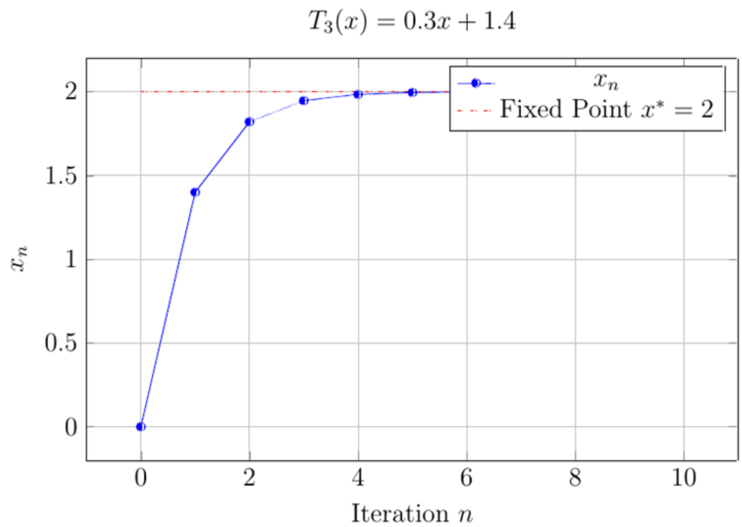

Figure 3: Plot of the iterative sequence x_n for $T_3(x) = 0.3x + 1.4$ with fixed point $x^* = 2$.

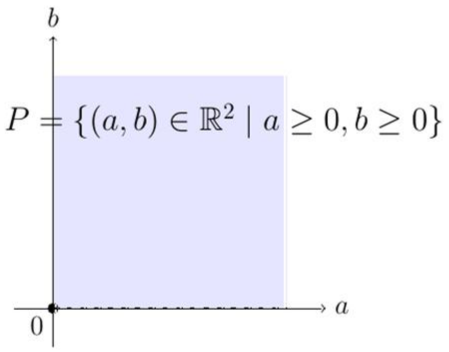

Figure 4: Metric cone P in $\mathbb{R}^2$ defined by nonnegative a and b.

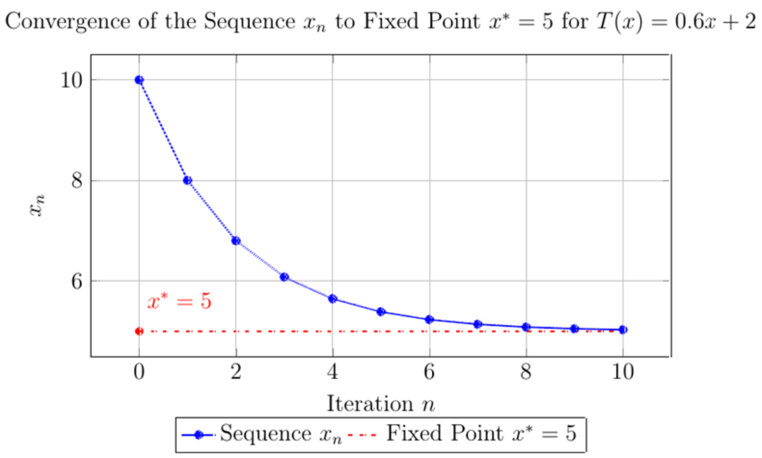

Figure 5: Graph showing the convergence of the sequence x_n to the fixed point $x^* = 5$ generated by a generalized Lipschitzian map for the function $T(x) = 0.6x + 2$.

A Contraction Theorem On The Cone Metric Space $(\mathbb{R}, d)$

Theorem 1. Let $X = \mathbb{R}$ and define the cone metric $d: X \times X \to \mathbb{R}^2$ by $d(x, y) = |x - y|(1,1)$, with cone $P = \{(a, b) \in \mathbb{R}^2 : a \geq 0,\ b \geq 0\}$. Let $T(x) = kx + c$, where $k \in [0,1)$ and $c \in \mathbb{R}$. Then (1). T is a contraction on $(\mathbb{R}, d)$ with contraction constant k, that is, $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in \mathbb{R}$.
(2). T has a unique fixed point $x^* = \frac{c}{1-k}$.
(3). For any initial point $x_0 \in \mathbb{R}$ the Picard iteration $x_{n+1} = T(x_n)$, $n \geq 0$, converges to x^*. Moreover the error admits the exact formula $|x_n - x^*| = k^n|x_0 - x^*|$ and the cone-metric bound $d(x_n, x^*) \leq k^n d(x_0, x^*)$.
In particular the following useful estimate holds: $d(x_n, x^*) \leq \frac{k^n}{1-k} d(x_0, x_1)$, $n \geq 1$.

Proof 1.
(1) For any $x, y \in \mathbb{R}$, $d(Tx, Ty) = |kx + c - (ky + c)|(1,1) = |k||x - y|(1,1) = kd(x, y)$, so T is a contraction with constant $k \in [0,1)$.

(2) A fixed point x^* satisfies $x^* = kx^* + c$. Solving for x^* (since $1 - k > 0$) gives $x^* = \frac{c}{1-k}$, and uniqueness follows from the contraction property: if y^* were another fixed point then
$d(x^*, y^*) = d(Tx^*, Ty^*) \leq kd(x^*, y^*)$ so $(1 - k)d(x^*, y^*) \leq 0$ which follows that
$d(x^*, y^*) = 0$, hence $x^* = y^*$.

(3) For the affine map one can compute the closed form for the iterates: $x_n = k^n x_0 + c\frac{1-k^n}{1-k}$.
Subtracting $x^* = \frac{c}{1-k}$ yields $x_n - x^* = k^n(x_0 - x^*)$, so $|x_n - x^*| = k^n|(x_0 - x^*)|$; therefore $d(x_n, x^*) = |x_n - x^*|(1,1) = k^n d(x_0, x^*)$, proving convergence since $k^n \to 0$ as $n \to +\infty$.

To obtain the bound with $d(x_0, x_1)$ note that $(x_0, x_1) = d(x_0, Tx_0) \geq (1 - k)\, d(x_0, x^*)$, which rearranges to $d(x_0 - x^*) \leq \frac{1}{1-k}\, d(x_0, x_1)$. Combining with $d(x_n, x^*) \leq k^n\, d(x_0, x^*)$ gives $d(x_n, x^*) \leq \frac{k^n}{1-k}\, d(x_0, x_1)$, $n \geq 1$, as required. This validation is similar to the results of Hardy and Rogers (1973) [17], Chidume (2009) [18]. The fixed point is unique for the contraction is strict (Rosler, 1992) [19] and Bharucha-Reid (1976) [20].

The specific maps used in the numerical examples satisfy the theorem. Hence:
(i). $T_1(x) = \frac{1}{2}x + 1$ has $k = \frac{1}{2}$ and fixed point $x^* = 2$.
(ii). $T_2(x) = 0.8x + 0.2$ has $k = 0.8$ and fixed point $x^* = 1$.
(iii). $T_3(x) = 0.3x + 1.4$ has $k = 0.3$ and fixed point $x^* = 2$.

For each map, the iteration $x_{n+1} = T_i(x_n)$ converges geometrically to the listed x^* with decay factor k.

V. Conclusion

The sequence x_n converges rapidly to the unique fixed point $x^* = 2$ of the contraction mapping $T_1(x) = \frac{1}{2}x + 1$, in accordance with the fixed point theorems for generalized Lipschitzian maps in cone metric spaces. The graph and simulation results presented in the work numerical simulation of iterative induced sequence on cone metric spaces explore the behavior of iterative sequences under Lipschitzian contraction mappings within cone metric spaces. The main focus is on demonstrating, through numerical simulation and plots, how iterative processes can converge to unique fixed points, thereby verifying the theoretical underpinnings of fixed point theorems in cone metric spaces.

A cone metric space generalizes the concept of traditional metric spaces by incorporating a real Banach space E and a subset P ⊂ E that satisfies certain properties to constitute a cone. In this study, the cone metric is defined on $\mathbb{R}$ with the distance function given by $d(x, y) = |x - y|.(1,1)$. Here, the cone $P \subset \mathbb{R}^2$ consists of vectors whose components are all non-negative.

The study investigated the convergence of iterative sequences generated by mappings of the form $T(x) = ax + b$, where the constants a and b ensure the function is a contraction specifically, where $0 < a < 1$. According to the Banach fixed point theorem (extended to cone metric spaces), any such contraction mapping on a complete metric space has a unique fixed point, and the iteration $x_{n+1} = T(x_n)$ converges to this point.

For each mapping, the iterative sequence $\{x_n\}$ starts from $x_0 = 0$ and evolves by repeatedly applying the mapping. The sequences are then plotted across 11 iterations.

The graphical plots clearly illustrate convergence behaviour:
i. For $T_1(x)$, the sequence rapidly approaches the fixed point $x^* = 2$.
ii. For $T_2(x)$, convergence to $x^* = 1$ is slower due to the higher contraction constant.
iii. For $T_3(x)$, convergence to $x^* = 2$ is very rapid due to the smaller contraction constant.

The dashed red line in each graph marks the fixed point, while the blue points represent the iterative values x_n. As expected from contraction mapping theory, all sequences move monotonically toward their respective fixed points.

The diagram above in the Figure 4 represents the metric cone P in the Euclidean plane $\mathbb{R}^2$, specifically defined as: $P = \{(a, b) \in \mathbb{R}^2: a \geq 0, b \geq 0\}$

This set includes all points in the first quadrant where both coordinates are non-negative. In the visualization:
(i). The horizontal axis is labeled a, and the vertical axis is labeled b.
(ii). The blue shaded region indicates the cone P, representing all points where $a \geq 0$ and $b \geq 0$. This includes the positive directions of both axes starting from the origin.
(iii). Dashed lines from the origin to the points on the axes highlight the boundaries of the cone.
(iv). The origin, denoted as 0, is marked with a dot to show the vertex of the cone.
(v). A textual label within the shaded region explicitly states the definition of P.

This visualization helps to conceptualize the metric cone as a subset of $\mathbb{R}^2$ that forms a convex region extending infinitely in the positive a and b directions.

The Figure 5 illustrates the convergence of the sequence $\{x_n\}$ generated by a generalized Lipschitzian map $T(x) = 0.6x + 2$. Starting from $x_0 = 10$, the sequence rapidly approaches the fixed point $x^* = 5$, demonstrating stability and contraction under the Lipschitz condition.

The study confirms that contraction mappings defined on cone metric spaces yield sequences that converge to unique fixed points. The speed of convergence is influenced by the magnitude of the contraction constant k, with smaller values of k promoting faster convergence. These simulations offer visual and numerical support for fixed point theorems in cone metric spaces and demonstrate their potential in numerical analysis and computational mathematics.

Declaration Statements

i. **Author's Contributions:** All authors contributed equally to the conception, development and preparation of this article.

ii. **Conflicts of Interest/ Competing Interests:** The authors declare that there are no conflicts of interest associated with this article.

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