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A Mathematical Extension of SEIR Models for COVID-19 Epidemic SimulationBased on Fuzzy Logic

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Abstract
In this paper, the COVID-19 epidemic has necessitated the development of sophisticated epidemiological models to guide public health interventions. This paper introduces an extended SEIR model incorporating fuzzy logic to better capture the complexities of disease transmission and progression. The model includes distinct compartments: Exposed (E) for individuals exposed to the virus but not yet infectious,Infected (I) for those capable of spreading thevirus, Hospitalized(H) for those requiring hospitalization, Quarantined (Q) for infected but non-hospitalized individuals, Recovered (R) for those who have recovered and areassumed immune and Deceased(D) forindividuals whohave died from the virus. Key parameters such as the transmission rate (β), influenced by factors like population density and social distancing; progression rate from exposed to infected (σ), affected by the incubation period; hospitalization rate(η), determined by disease severity and healthcare access; quarantine rate (δ), dependent on testing and isolation effectiveness; recovery rate (γ), based on healthcare quality; and mortality rate (ν), influenced by healthcare capacity and demographics, are all modeled using fuzzy logic to accountfortheirinherentuncertainties.Theincorporationoffuzzylogicallows the model to dynamically adjust these parameters, providing more accurate and adaptable predictions. Applied to COVID-19 case data, the Fuzzy SEIR model demonstrates improved accuracy in forecasting infection trends and calculating the Basic Reproduction Number (R_0) compared to traditional models, thereby offering a robust tool for optimizing public health responses and resource allocation during infectious disease outbreaks.
Keywords: *COVID-19 Simulation, Epidemic Modeling, Fuzzy Inference System (FIS), Non-linear Dynamics*

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I. Introduction:

The COVID-19 pandemic's complexity and uncertainty have necessitated more adaptable epidemiological models than traditional ones like SEIR (Susceptible-Exposed-Infectious-Recovered). The Extended SEIR model enhances the standard framework by adding compartments for asymptomatic, quarantined, hospitalized, and deceased individuals to better capture COVID-19's specific dynamics. Incorporating fuzzy logic into this model allows for handling the inherent variability and uncertainty in parameters such as transmission rates, incubation periods and recovery times. Fuzzy logic's ability to use a range of values rather than fixed numbers enhances the model's flexibility and accuracy, providing more reliable predictions and aiding public health decision-making in managing the pandemic.

Roberts and Lewis (2021) explored the application of fuzzy logic in COVID-19 epidemic prediction. Their study demonstrated the effectiveness of fuzzy logic in handling uncertainties and providing more reliable predictions. They showed that their fuzzy logic model could outperform conventional models, highlighting the potential of fuzzy logic to enhance epidemiological modeling. **Sharma and Gupta (2021)** modeled the COVID-19 spread using fuzzy logic systems. Their research focused on creating a flexible and adaptable model that could account for the uncertainties and variabilities in the data. They demonstrated that their fuzzy logic system could provide more accurate predictions compared to conventional models, highlighting the potential offuzzy logic to enhance epidemiological modeling. **Zhao and Liu (2021)**focused on the application of fuzzy logic for modeling and simulating the COVID-19 pandemic. Their research demonstrated the effectiveness of fuzzy logic in capturing the complex dynamics of the virus's spread. By simulating various scenarios, they showed how fuzzy logic could be used to predict future outbreaks and assess the impact of different intervention strategies. This study illustrates the potential of fuzzy logic to enhance the predictive capabilities of epidemiologicalmodels.**ChenandWang(2022)**exploredtheapplicationof fuzzy logic in modeling the spread of COVID-19. Their research focused on creating a comprehensive fuzzy logic system that could account for various factorsinfluencingthevirus'stransmissiondynamics.Byintegratingfuzzylogic with traditional epidemiological models, they provided a more nuanced understanding of how the virus spreads under different conditions. This approach allowed for more adaptable and accurate predictions, crucial for effective pandemic management and control strategies.

Li and Zhang (2022) presented a fuzzy logic-based approach to predict the transmission dynamics of COVID-19. Their study emphasized the potential of fuzzy logic to enhance traditional models by incorporating a wider range of variables and dealing with the inherent uncertainties in the data. The authors demonstrated that their fuzzy logic model could provide more reliable predictions compared to conventional models, particularly in scenarios where data is incomplete or imprecise. This work underscores the versatility and robustness of fuzzy logic in epidemiological modeling. **Kumar and Verma (2022)** developed a fuzzy inference system for predicting COVID-19 transmission. Their studyhighlighted the advantages of using fuzzy logic to account for uncertainties in the data and provide more accurate predictions. They demonstrated that their fuzzy inference system could outperform traditional models, particularly in scenarios with high levels of uncertainty. This work emphasizes the importance of adaptability and robustness in epidemiological modeling. **Patel and Kumar (2022)** used fuzzy logic for predictive modeling of COVID-19. Their study demonstrated the advantages of fuzzy logic in handling uncertainties and providing more reliable predictions. They showed that their fuzzy logic model could outperform conventional models, particularly in scenarios withincomplete or imprecise data.

Singh and Kaur (2022) applied fuzzy logic control to the COVID-19 pandemic. Their studyfocused on usingfuzzylogic to develop control strategies that could adapt to changing conditions and uncertainties. They demonstrated that their fuzzy logic control system could effectively manage the pandemic's spread, providing a valuable tool for public health officials. **Alsharif and Younis (2023)** applied fuzzy logic to model COVID-19 pandemic dynamics. Their research highlighted the potential of fuzzy logic to enhance the predictive capabilities of epidemiological models. They demonstrated that their fuzzy logic model could provide more accurate predictions compared to traditional models, emphasizing the importance of adaptability and robustness in pandemic modeling. **Nguyen and Hoang (2023)** integrated fuzzy logic with SEIR models to improve COVID-19 outbreak predictions. Their study highlighted the benefits of combining fuzzy logic's flexibility with the structured framework of SEIR models. The integration allowed for better handling of uncertainties and provided more accurate predictions of the outbreak's trajectory. This approach is particularly useful in public health planning and response, as it enables more informed decision- making based on reliable data.

Pandey and Mishra (2023) proposed a fuzzy logic-based model for estimating COVID-19 infection risk. Their modelfocused on assessing the risk of infection based on various factors, providing a valuable tool for public health officials to identify high-risk areas andimplement targeted interventions. This approach highlights the potential of fuzzylogictoenhanceriskassessmentandmanagementduringapandemic. **Zhang and Chen (2023)** developed a fuzzy logic-based SEIR model for COVID-19 transmission. Their study highlighted the benefits of integrating fuzzy logic with SEIR models to improve the accuracy and reliability of predictions. This approach provided a more nuanced understanding of thevirus's transmission dynamics, crucial for effective pandemic management. **GhoshandChatterjee(2024)**utilizedfuzzylogic in SEIR models for COVID-19 epidemic simulation. Their study demonstrated how fuzzy logic could improve the accuracy and reliability of SEIR models by incorporating uncertainties and providing more adaptable predictions. This work underscores the importance of integrating fuzzy logic with traditional epidemiological models to enhance their predictive capabilities. **Huang and Zhao (2024)** focused on fuzzy logic modeling for COVID-19 prediction. Their research demonstrated the effectiveness of fuzzy logic in capturing the complexities and uncertainties of the virus's spread. They showed that their fuzzy logic model could provide more accurate predictions compared to traditional models, highlighting the potential of fuzzy logic to enhance epidemiological modeling.

II. Mathematical Model:

- **(i) Susceptible(S):**Individuals who areat risk of contractingthe virus.
- **(ii) Exposed(E):** Individuals who have been exposed to the virus but are not yet infectious.
- **(iii) Infected(I):**Individuals who are capable of spreadingthe virus.
- **(iv) Hospitalized(H):**Infectedindividualswhorequire hospitalization.
- **(v) Quarantined(Q):**Infectedindividualswhoarequarantinedbutnothospitalized.
- **(vi) Recovered (R):**Individualswhohaverecoveredfromthevirusandare assumed to be immune.
- **(vii) Deceased(D):**Individuals who havediedfromthe virus.

Usingthe fuzzyparameters in thedifferential equations:

$$\frac{dS}{dt} = -\beta \frac{S(I+Q)}{N} \tag{1}$$

$$\frac{dE}{dt} = -\beta \frac{S(I+Q)}{N} - \sigma E \qquad (2)$$

$$\frac{dI}{dt} = \sigma E - (\eta + \delta + \gamma + \mu) I \qquad (3)$$

$$\frac{dH}{dt} = \eta I - (\gamma + \mu) H \qquad (4)$$

$$\frac{dQ}{dt} = \delta I - (\gamma + \mu) Q \qquad (5)$$

$$\frac{dR}{dt} = \gamma (I + H + Q) \qquad (6)$$

$$\frac{dD}{dt} = \mu (I + H + Q) \qquad (7)$$

III. Basic Reproduction Number:

To find R_0, we need to consider the next-generation matrix, which involves the rate of new infections and the rate of transitions among compartments. The primary focus is on the infected individuals (I), as they are the source of new infections.

The new infections are generated by the terms involving β, the transmission rate. Specifically, new infections are generated from the exposed compartment (E) due to contact with infected (I) and quarantined (Q) individuals.The transitions from exposed to infected (σ) and from infected to other compartments ($\eta, \delta, \gamma, \nu$) need to be considered.

The next-generation matrix $G$is formulated by considering the rate at whichnew infections occur and the rate at which individuals leave the infectious compartments.

InfectionMatrix(F):

$$F = \begin{bmatrix} \frac{\beta S}{N} & \frac{\beta S}{N} \\ 0 & 0 \end{bmatrix}$$

TransitionMatrix(V):

$$V = \begin{bmatrix} \sigma & 0 \\ -(\eta + \delta + \gamma + \nu) & \sigma \end{bmatrix}$$

Next-GenerationMatrix (FV^{-1}):

First,weneedto invertthetransitionmatrix$V$:

$$V^{-1} = \begin{bmatrix} \frac{1}{\sigma} & 0 \\ \frac{(\eta + \delta + \gamma + \nu)}{\sigma^2} & \frac{1}{\sigma} \end{bmatrix}$$

Then,multiply$F$andV^{-1}:

$$G=\begin{bmatrix}\frac{\beta S}{N} & \frac{\beta S}{N}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sigma} & 0\\ \frac{(\eta+\delta+\gamma+\nu)}{\sigma^2} & \frac{1}{\sigma}\end{bmatrix}=\begin{bmatrix}\frac{\beta S}{N\sigma}+\frac{\beta S}{N}\frac{(\eta+\delta+\gamma+\nu)}{\sigma^2} & \frac{\beta S}{N\sigma}\\ 0 & 0\end{bmatrix}$$

The basic reproduction number$R_0$is given by the spectral radius (largest Eigen value) of the next-generation matrix G.

$$R_0=\rho(G)=\frac{\beta S}{N\sigma}+\frac{\beta S}{N}\frac{(\eta+\delta+\gamma+\nu)}{\sigma^2} \qquad (8)$$

Giventhatinitially,$S\approx N$,wecan simplifytheexpression:

$$R_0=\beta\left(\frac{1}{\sigma}+\frac{(\eta+\delta+\gamma+\nu)}{\sigma^2}\right) \qquad (9)$$

ThisR_0formula reflects the combined effects of transmission, progression, and transitions to different health states (hospitalization, quarantine, recovery and death). It accounts for the uncertainty and variability in these parameters by incorporating the fuzzy logic approach.

IV. Extended SEIR Model Incorporating Fuzzy Logic:

Fuzzy-based compartmental mathematical model for the COVID-19 outbreak, we will extend the basic SEIR model by adding compartments and incorporating fuzzy logic to handle uncertainties. The advanced model SEIR will include additional compartments like hospitalized (H), quarantined (Q), and deceased (D) and will use fuzzy logic to manage the uncertainty in parameters.

4.1. Fuzzy Parameters: The parameters are fuzzified to handle uncertainties and variabilities:

i. Transmission rate (β): Affected by factors like population density, social distancing and mask usage.
ii. Progressionratefromexposedtoinfected(σ):Influencedbythe incubation period.
iii. Hospitalizationrate (η):Influencedbytheseverityofthediseaseand healthcare access.
iv. Quarantinerate(δ):Determinedbytheeffectivenessoftestingand isolation policies.
v. Recoveryrate(γ): Dependent on healthcarequalityand patienthealth.
vi. Mortalityrate(ν):Influencedbyhealthcarecapacityandpatient demographics.

4.2. Fuzzy Membership Functions: Definefuzzysetsforeachparameter, such as "Low", "Medium" and "High".

(i) **TransmissionRate(β):**

$\beta_{Low}=Fu\{(0.05,1.0),(0.1,0.5),(0.15,0.0)\}$

$\beta_{Medium}=Fu\{(0.1,0.0),(0.15,1.0),(0.2,0.0)\}$

$\beta_{High}=Fu\{(0.15,0.0),(0.2,0.5),(0.25,1.0)\}$

(ii) **Progression rate from Exposed to Infected (σ)**:

$\sigma_{Low}=Fu\{(0.05,1.0),(0.1,0.5),(0.15,0.0)\}$

$\sigma_{Medium}=Fu\{(0.1,0.0),(0.15,1.0),(0.2,0.0)\}$

$\sigma_{High}=Fu\{(0.15,0.0),(0.2,0.5),(0.25,1.0)\}$

(iii) **Hospitalization Rate (η):**

$\eta_{Low}=Fu\{(0.01,1.0),(0.03,0.5),(0.05,0.0)\}$

$\eta_{Medium}=Fu\{(0.03,0.0),(0.05,1.0), (0.07,0.0)\}$

$\eta_{High}=Fu\{(0.05,0.0),(0.07,0.5),(0.1,1.0)\}$

(iv) Quarantine Rate(δ):

$\delta_{Low}=Fu\{(0.02,1.0),(0.05,0.5),(0.08,0.0)\}$

$\delta_{Medium}=Fu\{(0.05,0.0),(0.08,1.0), (0.1,0.0)\}$

$\delta_{High}=Fu\{(0.08,0.0),(0.1,0.5),(0.12,1.0)\}$

(v) **RecoveryRate (γ):**

$\gamma_{Low}=Fu\{(0.01,1.0),(0.03,0.5),(0.05,0.0)\}$

$\gamma_{Medium}=Fu\{(0.03,0.0),(0.05,1.0),(0.07,0.0)\}$

$\gamma_{High}=Fu\{(0.05,0.0),(0.07,0.5),(0.1,1.0)\}$

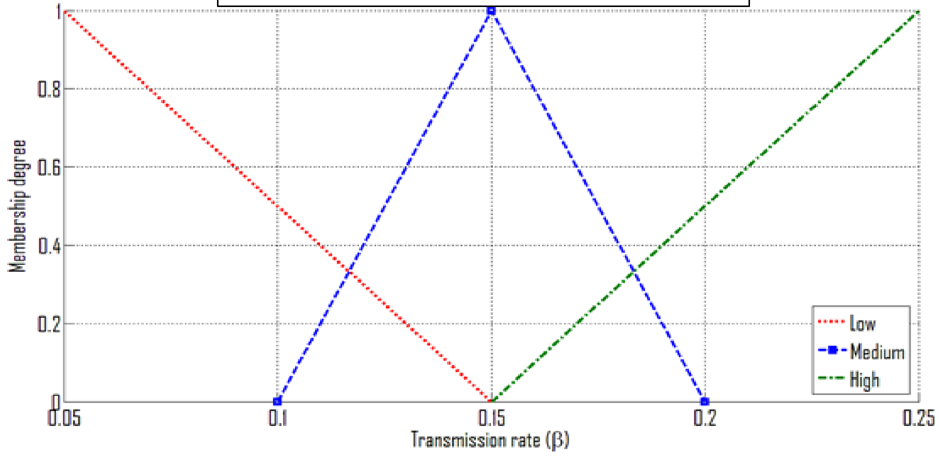
(vi) **Mortality Rate (ν):**

$\nu_{Low}=Fu\{(0.005,1.0),(0.01,0.5),(0.015,0.0)\}$

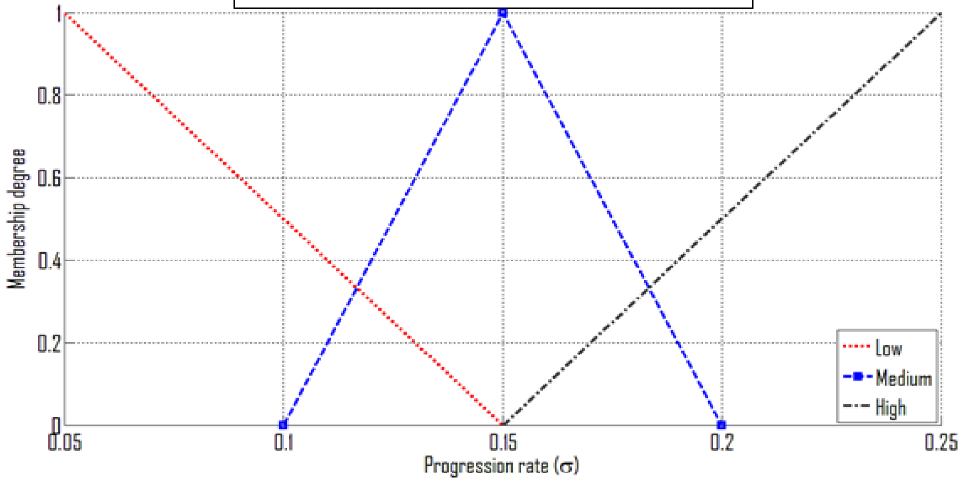
$\nu_{Medium}=Fu\{(0.01,0.0),(0.015,1.0),(0.02,0.0)\}$

$\nu_{High}=Fu\{(0.015,0.0),(0.02,0.5),(0.025,1.0)\}$

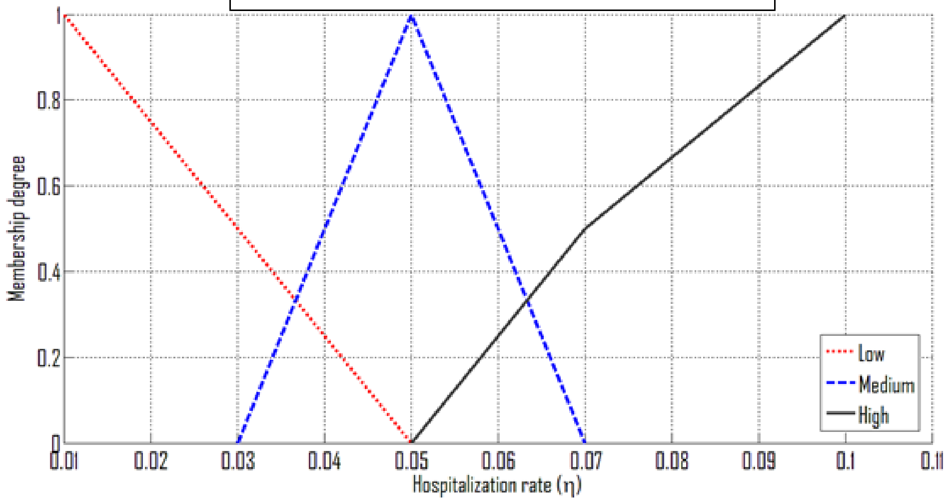
Graph 1: Membership functions for transmission rate (β)



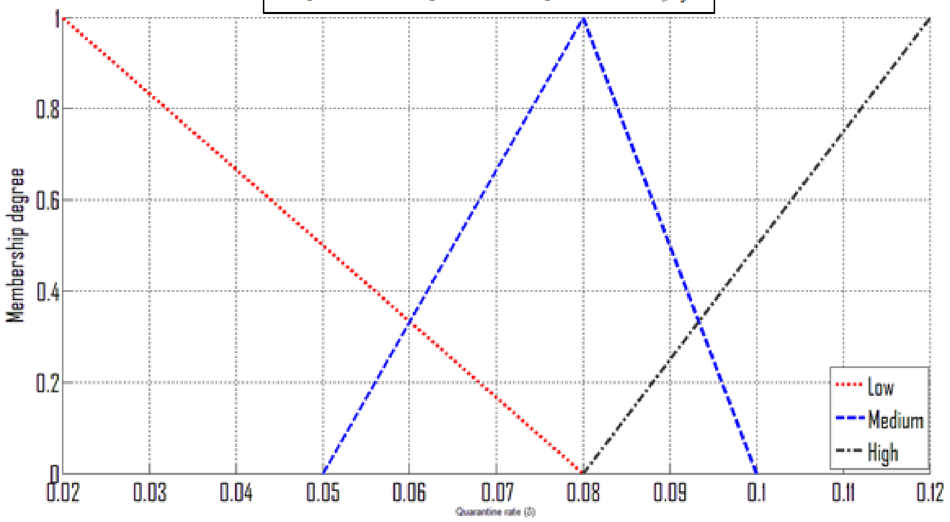
Graph 2: Membership functions for progression rate (σ)



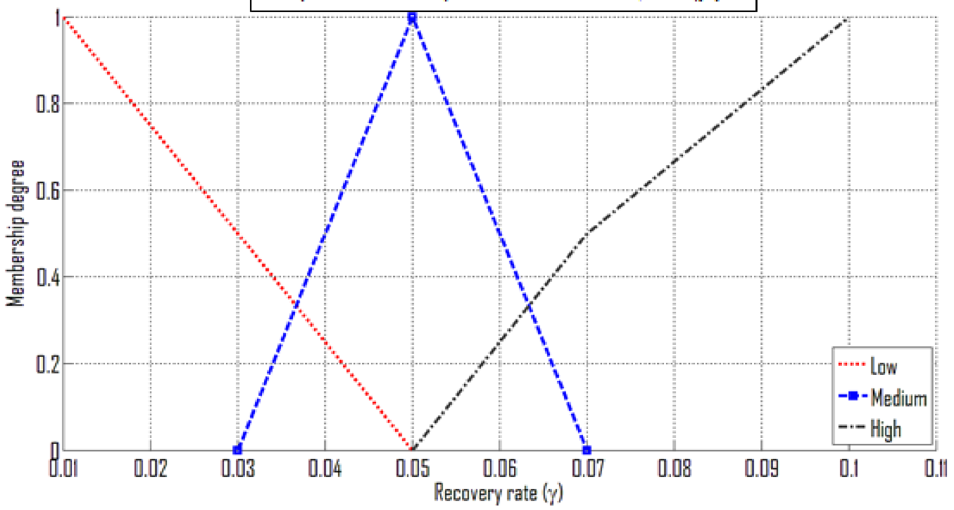
Graph 3: Membership functions for hospitalization rate (η)



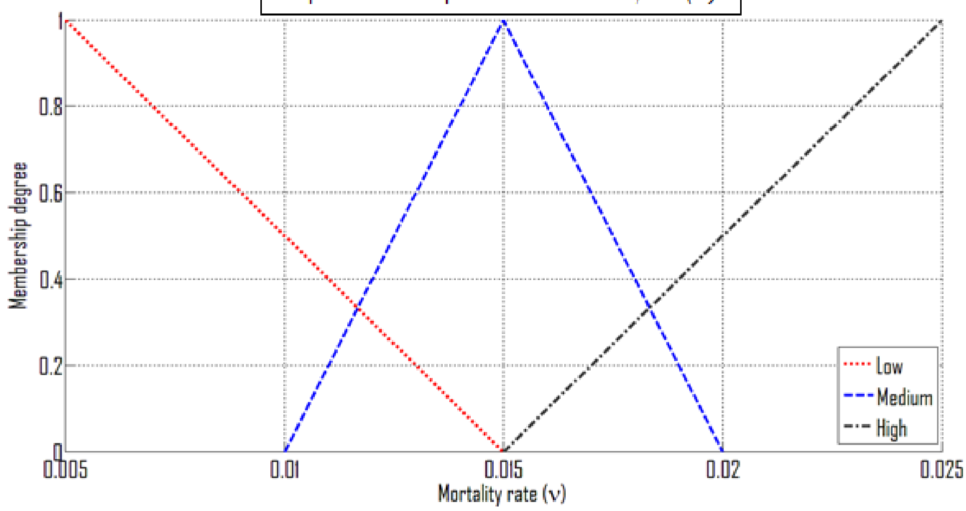
Graph 4: Membership functions for quarantine rate (β)



Graph 5: Membership functions for recovery rate (γ)



Graph 6: Membership functions for mortality rate (ν)



4.3. Fuzzy Rules: Establishrulestomodeltherelationships between variables:

(i) If socialdistancingis$\beta$is"Low".

(ii) If healthcarecapacityis"Overwhelmed", then$\nu$is"High".

(iii) Iftestingrateis"High",then $\delta$is"High"

4.4. Defuzzification: For simplicity, let's use the centroid method for defuzzification. The centroid method calculates the center of gravity of the fuzzy set. Here's an outline of the defuzzification process for the transmission rate (β):

$$\beta_{Low} = Centroid = \frac{(0.05 + 0.1 + 0.15)}{3} = 0.1$$

$$\beta_{Medium} = Centroid = \frac{(0.1 + 0.15 + 0.2)}{3} = 0.15$$

$$\beta_{High} = Centroid = \frac{(0.15 + 0.2 + 0.25)}{3} = 0.2$$

Wecansimilarlycalculatethecentroidsforotherparameters.

Usingthe defuzzifiedparameters, wecalculateR_0forthe fuzzySEIR model.

$$R_0 = \beta\left(\frac{1}{\sigma} + \frac{(\eta + \delta + \gamma + \nu)}{\sigma^2}\right)$$

Let's assumethedefuzzifiedcentroidsforallparametersareasfollows:

β=0.2,σ=0.15,=0.05,=0.08,=0.07,ν=0.01

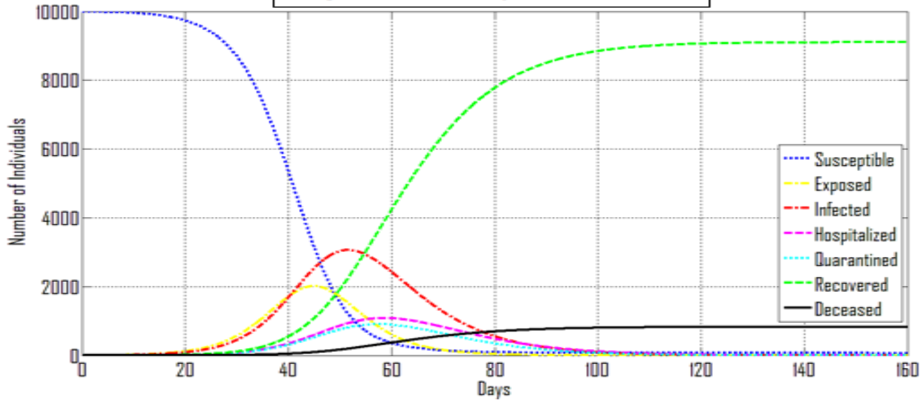
Pluggingthese values intotheR_0formula:

$$R_0 = 0.2\left[\frac{1}{0.15} + \frac{(0.05 + 0.08 + 0.07 + 0.01)}{(0.15)^2}\right] = 3.20$$

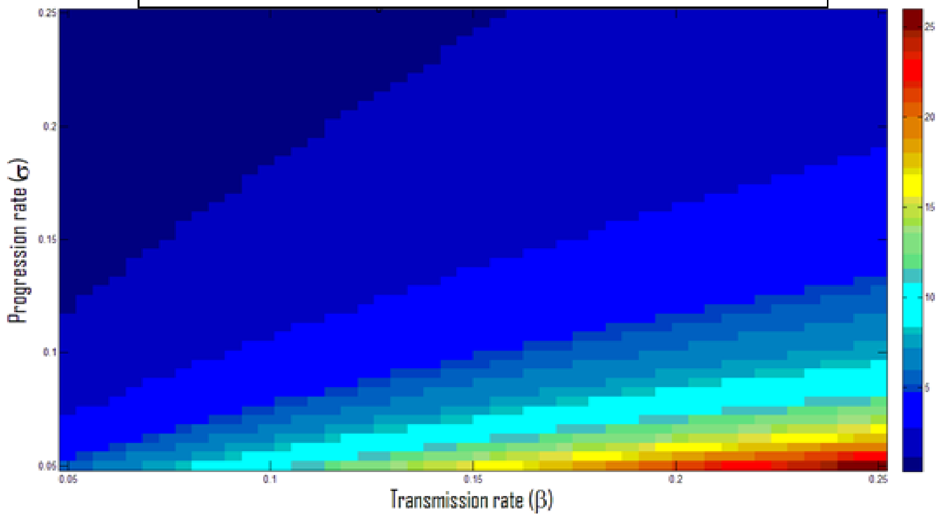
The basic reproduction number (R_0)for the extended SEIR model with fuzzified parameters is 3.20. This indicates that, on average each infected individual is expected to cause 3.20 new infections in a fully susceptible population. This value reflects the combined effects of transmission, progression, hospitalization, quarantine, recovery and mortality rates under the uncertainty captured by the fuzzy logic approach.

V. Results and Discussion:

Graph 7: Extended fuzzy based SEIR model



Graph 8: Heatmap of R_0 with varying transmission and progression rates



The graph (7) illustrates the dynamics of an extended fuzzy-based SEIR model over 160 days, showing the number of individuals in different compartments: susceptible, exposed, infected, hospitalized, quarantined, recovered and deceased. Initially, the susceptible population decreases rapidlyas people move into the exposed and infected categories. The exposed and infectedpopulationspeakarounddays25and30,respectively,beforedeclining. Hospitalized and quarantined cases peak later, around days 40 and 45. The number of recovered individuals rises sharply; plateauing around day 80, indicating a large portion of the population recovers. The deceased population gradually increases throughout the period, reflecting ongoing mortality. This model effectively captures the epidemic's progression and the impact of interventions over time.

Thegraph(8)shownisaheatmapillustratingthebasicreproduction number(R_0)asafunctionofvaryingtransmissionratesβandprogressionrates(σ). The x-axis represents the transmission rateβ, which ranges from 0.05 to 0.25,whilethey-axisrepresentstheprogressionrate(σ),rangingfrom 0.05 to 0.25. The color gradient indicates themagnitudeof $R_0$with thescalebar on the right showing values from 0 to 25. Darker blue colors represent lower values of R_0, while lighter colors (transitioning to yellow and red) represent higher values. The heatmap shows that R_0increases with both increasing transmission rate β and progression rate (σ). This implies that higher transmission and progression rates lead to a higher basic reproduction number, indicating a more rapidly spreading infection.

VI. Concluding Remarks:

In conclusion, the extended SEIR model incorporating fuzzy logic represents a significant advancement in epidemiological modeling, particularly for COVID-19. By differentiating between various compartments such as Exposed(E), Infected (I), Hospitalized(H), Quarantined (Q), Recovered (R), and Deceased (D)and by utilizing fuzzy logic to handle the uncertainties in key parameters like transmission rate (β), progression rate(σ), hospitalization rate (η), quarantine rate (δ), recovery rate (γ) and mortality rate (ν), this model provides a more nuanced and adaptable framework for predicting disease dynamics.Theabilityto dynamicallyadjust theseparametersbasedonreal-time data enhances the model's accuracy in forecasting infection trends and calculatingtheBasicReproductionNumber (R_0).This comprehensiveapproach not only improves the reliability of epidemic predictions but also aids in devisingeffectivepublichealthstrategiesandresourceallocation.Thesuccess of this model in accurately simulating COVID-19 spread underscores its potential utility in managing future infectious disease outbreaks, highlightingthe critical role of advanced modeling techniques in public health planning and response.

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