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A Stochastic Compartmental Model of Influenza with Birth and Awareness Effects

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Abstract
Background: *Influenza is an infectious viral disease of the upper respiratory tract caused by the orthomyxoviradae viruses, which occurs in subtypes A or B, depending on their core protein.The disease has always been a major issue of health concern in Nigeria and many parts of Africa and Asia with a yearly increase in the number of susceptible, infected and dead persons. Most mathematical models on the disease often employ the popular deterministic SIR (Susceptible- Infective-Removed) model. Little or no attention has been given to the national birth rate and death rate uncertainties which are important parameters for influenza dynamics. Hence the aim of this study is to examine the relative impact of birth and awareness rates on the size of the susceptible classes of the population.*
Materials and Methods: *In this study we have extended the SIR modelling to a four-compartment SSIR (Susceptible-Susceptible-Infective-Removed) model with two Susceptible compartments (Informed and Uninformed) describing the interaction and impact of birth, death and awareness rates of influenza on the susceptible class of the population. The model consists of a system of four stochastic differential equations (SDEs) which are linear in the narrow sense, that is, with additive noise. The SDEs were solved using Ito's formula and analysed with secondary data from both private researchers and organisations .*
Results: *Analysis of the sample paths of the two susceptible classes showed that, the two susceptible classes keep rising in size with increased birth rate in spite of awareness campa*ign,
Conclusion: *High birth rate is major contributory factor to influenza sprea*d.
Key Word: *Informed class; Uninformed class;, Removed class; Brownian motion; Ito's formula*

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I. Introduction

The global mortality of influenza world-wide is enormous. 'Influenza causes between 250,000 and 500,000 deaths world-wide out of 3 to 5 million cases of severe illness annually' (Lagere et al. (2020). According to the world health ranking by World Health Organisation (WHO, 2020), Nigeria is among the most influenza endemic nations in the world.

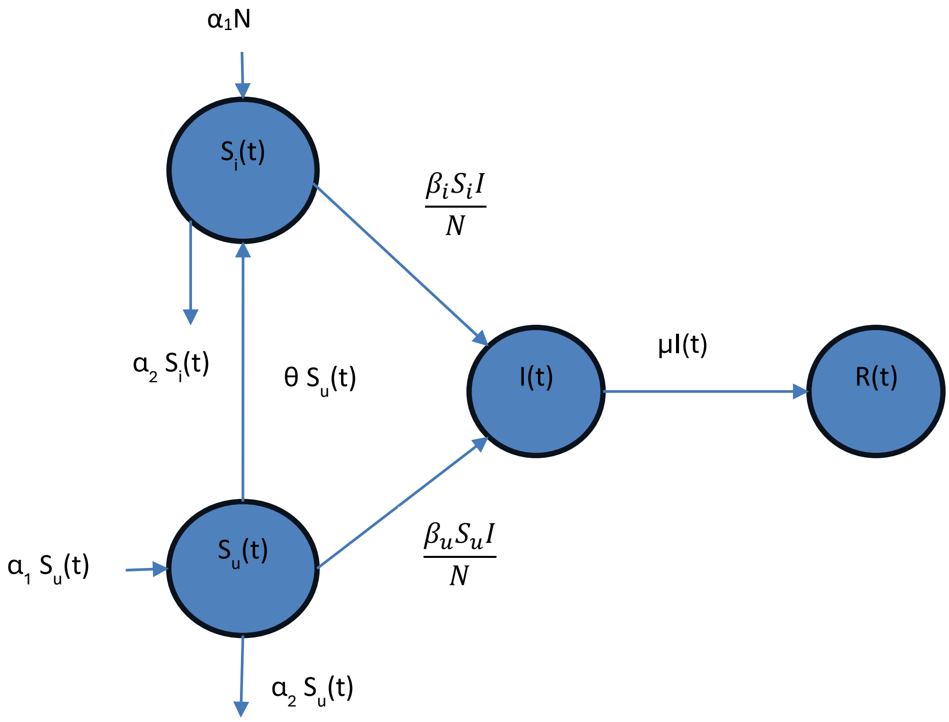
In the study of influenza prevention, transmission and control, the deterministic SIR model has always been a popular tool. For example, Alcaraz and Vargas (2020) used the deterministic SIR model to study the control measures for influenza A HN1 epidemic. They found that an increase in vaccination rate results in decrease in reproduction number (R_O,) which is a threshold parameter for epidemic outbreak.

Demir and Vural (2024) used the SIR model to study influenza transmission by separating the susceptible class S into three sub-classes namely: Vaccinated and Protected, Vaccinated and Unprotected and Unvaccinated. Infected people were separated into Treated and Untreated. Using Caputo fractional derivative, they found that vaccinating the susceptible people significantly reduces influenza transmission. McKendrick (1926) was one of the earliest people to introduce randomness into a deterministic model to develop a continuous-time stochastic epidemic model. This was later followed by other works like the 1928 chain-binomial model of Reed and Frost as cited by Jacquez (1987); Bartlett (1949) and Kurtz (1978). More recently, we have Logini (2005), Yajiet al. (2007). Zhao et al.(2019) and Ferah et al.(2020). Stochastic models are now more frequently used in mathematical modeling of infectious diseases because of the incorporation of uncertainty which is a part of most real-life systems.

The objectives of the research are: (i) To model each of the four compartments of the population by a stochastic differential equation, (ii) To solve the SDEs modelling the susceptible classes analytically, (iii) To use the sample paths in analysing the variations in the sizes of the two susceptible classes.

II. Materials and Methods

Fig. 1: A Schematic Representation of the Compartmental Model



As shown in Figure 1 we adopt an extended form of SIR epidemic model called SSIR consisting of two susceptible classes, one infective class and one removed class. To study the relative impact of information spread rate and birth rate on the stochastic behaviour of the susceptible compartments of the SSIR model, we have classified the susceptible category into Informed class S_i and Uninformed class S_u .

A member of the population is in S_i at time t by birth at the rate α_1 or by transition of an uninformed person who is now aware of influenza preventive measures at the rate θ. A person can leave S_i by deaths other than influenza at the rate α_2 or by becoming infected with influenza virus and moving to Infective class I(t) at Poisson rate β_i. Infection is assumed to occur immediately a contact is made.

The uninformed compartment can increase only by birth at the rate α_1 but can decrease when an uninformed person becomes informed and moves to S_i at the rate θ or becomes infected and migrates to the infective compartment at the rate β_u

At the Infective compartment I(t), increases occur in two ways: (i) by infection of an S_i member at the rate β_i, (ii) by infection of an S_u member at the rate β_u. A decrease in I(t) occurs at the Poisson rate μ by removal to R(t). This is due to treatment of infected person, vaccination, quarantine or death or any means by which a member of the population is a zero contributor to the spread of influenza.

From the stochastic SIR model of (Bartlett, 1949), the stochastic rate of infection is $\frac{\beta SI}{N}$, where the β is the infection rate or stochastic intensity. The stochastic rate of removal is μI.

Thus, the expected changes in the 4 compartments in small time interval Δt are.

$E(\Delta S_i(t)) = (\alpha_1 N - \alpha_2 S_i + \theta S_u - \frac{\beta_i S_i I}{N})\Delta t$ (1)

$E(\Delta S_u(t)) = (\alpha_1 N - \alpha_2 S_u - \theta S_u - \frac{\beta_u S_u I}{N})\Delta t$ (2)

$E(\Delta I(t)) = (\frac{\beta_i S_i I}{N} + \frac{\beta_u S_u I}{N} - \alpha_2 I - \mu I)\Delta t$ (3)

$E(\Delta S(t)) = \mu I \Delta t$ (4)

The actual change in the Informed Susceptible compartments, taking into account the centred Poisson increment arising from birth and death in S_i, S_u and I, are

$\Delta S_i(t) = (\alpha_1 N - \alpha_2 S_i + \theta S_u - \frac{\beta_i S_i I}{N})\Delta t + \Delta Z_1 + \Delta Z_2 - \Delta Z_3$ (5)

$\Delta S_u(t) = (\alpha_1 N - \alpha_2 S_u - \theta S_u - \frac{\beta_i S_u I}{N})\Delta t + \Delta Z_2 - \Delta Z_1 - \Delta Z_3$ (6)

Where ΔZ_k, k=1, 2, 3, 4 are normally distributed with mean 0 and variances $(\alpha_1 N - \alpha_2 S_i + \theta S_u)\Delta t$, $(\alpha_1 N - \alpha_2 S_u - \theta S_u)\Delta t$ and $(\frac{\beta_i S_i I}{N} + \frac{\beta_u S_u I}{N} - \alpha_2 I)\Delta t$and $\mu I$for equations (1) to (4)) respectively, (Greenwood and Gordillo, 2009).
For N large, the Markov jump model is normalized by expressing the state variables as a proportion of the expected population E(N(t)) = N, Dividing through equations (5) and (6) by Δt and taking limits as Δt tends to 0.

$\frac{d}{dt}(\frac{S_i}{N}) = (\alpha_1 N - \alpha_2 \frac{S_i}{N_i} + \theta \frac{S_u}{N} - \beta_i \frac{S_i}{N}\frac{I}{.N}) + \sigma_1 \frac{dW_1}{dt} + \sigma_2 \frac{dW_2}{dt} - \sigma_3 \frac{dW_3}{dt} \Delta W_3$ (7)

$\frac{d}{dt}(\frac{S_u}{N}) = (\alpha_1 N - \alpha_2 \frac{S_u}{N_i} - \theta \frac{S_u}{N} - \beta_u \frac{S_u}{N}\frac{I}{.N}) + \sigma_2 \frac{dW_2}{dt} W_2 - \sigma_1 \frac{dW_1}{dt} - \sigma_3 \frac{dW_3}{dt}$ (8)

Clearing equations (7) and (8) of dt, since the Brownian motion is nowhere differentiable we let

$X_1 = \frac{S_i}{N}$, $X_2 = \frac{S_u}{N}$ and $X_3 = \frac{I}{.N}$
(9)

Replacing each Z_i by appropriate Brownian motion with the same standard deviation.

Thus,

$dX_1 = (\alpha_1 - \alpha_2 Y_1 + \theta Y_2 - \beta_i Y_1 Y_3)dt + G_1 dW_1 + G_2 dW_2 - G_3 dW_3$, $X_1(0) = x_i$ (10)

$dX_2 = (\alpha_1 - \alpha_2 Y_2 - \theta Y_2 - \beta_u Y_2 Y_3) + G_2 dW_2 - G_1 dW_1 - G_3 dW_3$, $X_2(0) = x_2$ (11)

The dynamics of a compartment depends on the initial sizes of other compartments linked to it.
Therefore, for dX_1, evaluate X_2 and X_3 at 0 and for dX_2, evaluate X_1 and X_3 at 0. Thus,

$dX_1 = [(\alpha_1 + \theta y_2) - (\alpha_2 + \beta_i y_3)X_1]dt + G_1 dW_1 + G_2 dW_2 - G_3 dW_3$, $X_1(0) = y_i$ (12)

$dX_2 = [\alpha_1 - (\alpha_2 + \theta + \beta_u y_3)X_2] + G_2 dW_2 - G_1 dW_1 - G_3 dW_3$, $X_2(0) = y_2$ (13)

Where $G_1 = \sqrt{(\alpha_1 - \alpha_2 y_1 + \theta y_2)}$, $G_2 = \sqrt{(\alpha_1 - \alpha_2 y_2 - \theta y_2)}$, $G_3 = \sqrt{(\beta_i y_1 I + \beta_u y_2 I - \alpha_2 I)}$

Equations (12) and (13) are autonomous linear SDEs in the narrow sense and with multi-dimensional Brownian motion.

Let $a_1 = \alpha_1 + \theta x_2$, $a_2 = -(\alpha_2 + \beta_i x)$, $\mathbf{V}_1 = (G_1, -G_2, -G_3)$ and $\boldsymbol{dW}^{(1)} = (W_1, dW_2, dW_3)$;

$b_1 = \alpha_1$, $b_2 = -(\alpha_2 + \theta + \beta_u y_3)$, $\mathbf{V_2} = (G_2, -G_1 - G_3)$ and $\boldsymbol{dW^{(2)}} = (W_2, dW_1, dW_3)$

The diffusion parts of eqs. (12) and (13) can be written as a scalar products so that

$$dY_1 = [a_1 + a_2X_1]dt + \mathbf{V_1}\boldsymbol{dW^{(1)}}, \; X_1(0) = x_i \tag{12}$$

$$dY_2 = [b_1 + b_2X_2]dt + \mathbf{V_2}\boldsymbol{dW^{(2)}}, \; X_2(0) = x_2 \tag{13}$$

Equation (12) is homogeneous when $a_1 = |\boldsymbol{V_1}| = 0$, so that $dX_1 = a_2X_1dt$ with the solution $\varphi_{t_0,t}(t) = e^{\int_{t_0}^{t} a_1(s)ds}$,

Similarly, eqn.(13) is homogeneous when $a_2 = |\boldsymbol{V_2}| = 0$; so, $dX_2 = a_2X_2dt$ with the solution $\varphi_{t_0,t}(t) = e^{\int_{t_0}^{t} a_1(s)ds}$

respectively known as the fundamental solutions of the SDEs (12) and (13), satisfying $\varphi_{t_0,t_0}^{-1} = 1$.

Using the Ito's formula as in Kloeden and Platen(1999); $dY = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dX + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}|\boldsymbol{V_i}|^2dt$, i = 1, 2, with the transformation $Y = f(t, X) = \varphi_{t_0,t}^{-1}X$,

Solving equation (12),

$dY = d(\varphi_{t_0,t}^{-1}X_1) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dX_1 + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}|\boldsymbol{V_1}|^2dt$ where the vector $\mathbf{V_1} = (G_1, -G_2, -G_3)$

$$= \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}((b_1 + b_2X_1)dt + G_1dW_1(t) - G_2dW_2(t) - G_3dW_3(t)) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}|\boldsymbol{V_1}|^2dt$$

$$= [\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}(b_1 + b_2X_1) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}|\boldsymbol{V_1}|^2]dt + \frac{\partial f}{\partial x}(G_1dW_1(t) - G_2dW_2(t) - G_3dW_3(t)$$

With $f(t, X(t)) = F_{t_0t}^{-1}X$, we have $\frac{\partial f}{\partial x} = F_{t_0t}^{-1}, \frac{\partial^2 f}{\partial x^2} = 0$

$$d(\varphi_{t_0,t}^{-1}X_1) = [\frac{\partial F_{t_0t}^{-1}}{\partial t}X_1\, \varphi_{t_0,t}^{-1}b_1 + \varphi_{t_0,t}^{-1}b_2X_1]dt + \varphi_{t_0,t}^{-1}(G_1dW_1(t) - G_2dW_2(t) - G_3dW_3(t)$$

But $F_{t_0t}^{-1} = e^{-\int_{t_0}^{t} b_2 ds}$, so $\frac{\partial F_{t_0t}^{-1}}{\partial t} = -b_2F_{t_0t}^{-1}$.

Therefore,

$$d(\varphi_{t_0,t}^{-1}X_1) = \varphi_{t_0,t}^{-1}b_1dt + \varphi_{t_0,t}^{-1}(G_1dW_1(t) - G_2dW_2(t) - G_3dW_3(t))$$

$$\varphi_{t_0,t}^{-1}X_1 = \varphi_{t_0,t}^{-1}X_1(0) + \int_{t_0}^{t}\varphi_{t_0,t}^{-1}b_1ds + \int_{t_0}^{t}G_1dW_1(s)ds - \int_{t_0}^{t}G_2dW_2(s)ds - \int_{t_0}^{t}G_3dW_3ds$$

Since $\varphi_{t_0,t_0}^{-1} = e^0 = 1$,

$\varphi_{t_0,t_0}^{-1}X_1$=$X_1(0)$+ $\int_{t_0}^{t}\varphi_{t_0,t}^{-1}b_1 ds$ + $\int_{t_0}^{t}\varphi_{t_0,t}^{-1}G_1 dW_1(s) + \int_{t_0}^{t}\varphi_{t_0,t}^{-1}G_2 dW_2(s) - \int_{t_0}^{t}\varphi_{t_0,t}^{-1}G_3 dW_3(s)$,

X_1 $= \varphi_{t_0,t}^{-1}(t)\{X_1(0)$ $+ \int_{t_0}^{t}\varphi_{t_0,t}^{-1}b_1 ds$ + $\int_{t_0}^{t}\varphi_{t_0,t}^{-1}G_1 dW_1(s)$ - $\int_{t_0}^{t}\varphi_{t_0,t}^{-1}G_3 dW_3(s)$

-$\int_{t_0}^{t}\varphi_{t_0,t}^{-1}G_4 dW_4(s)\}$

For $t_0 = 0$, $\varphi_{t_0,t}^{-1}$= $\exp(b_2 t)$ and $X_1(0) = x_1$

Therefore, the explicit solution of the SDE (12) is

$X_1 = e^{a_2 t}\left\{X_1(0) - \frac{a_1}{a_2}(e^{-a_2 t} - 1) + \int_{t_0}^{t} e^{-a_2 s} G_1 dW_1(s) + \int_{t_0}^{t} e^{-a_2 s} G_2 dW_2(s) - \int_{t_0}^{t} e^{-a_2 s} G_3 dW_3(s)\right\}$,
where $a_1 = \alpha_1 + \theta x_2$ $\quad a_2 = -(\alpha_2 + \beta_i x_3)$

Similarly

$X_2 = e^{b_2 t}\{X_2(0) - \frac{b_1}{b_2}(e^{-b_2 t} - 1) + \int_{t_0}^{t} e^{-bs} G_1 dW_1(s) - \int_{t_0}^{t} e^{-b_2 s} G_2 dW_2(s) - \int_{t_0}^{t} e^{-b_2 s} G_3 dW_3(s)\}$ where $b_1 = \alpha_1$ $\quad b_2 = -(\alpha_2 + \theta + \beta_u x)$

From the representations in eq. (9)

$S_i = Ne^{a_2 t}\{S_i(0) - \frac{a_1}{a_2}(e^{-a_2 t} - 1) + \int_{t_0}^{t} e^{-a_2 s} G_1 dW_1(s) + \int_{t_0}^{t} e^{-a_2 s} G_2 dW_2(s)$- $\int_{t_0}^{t} e^{-a_2 s} G_3 dW_3(s)\}$
where $a_1 = \alpha_1 + \theta x_2$ $\quad a_2 = -(\alpha_2 + \beta_i x_3)$

$S_u = e^{b_2 t}\{X_2(0) - \frac{b_1}{b_2}(e^{-b_2 t} - 1) + \int_{t_0}^{t} e^{-bs} G_2 dW_2(s) - \int_{t_0}^{t} e^{-b_2 s} G_1 dW_1(s) - \int_{t_0}^{t} e^{-b_2 s} G_3 dW_3(s)\}$,
where $b_1 = \alpha_1$ $\quad b_2 = -(\alpha_2 + \theta + \beta_u x)$

Estimation of Parameters and Analysis of Results

The Nigerian population, according to the United Nations Population Division, as at January 2022, is estimated to be 213 million. Thus, we shall fix our N at N = 213,000,000 . From the World Bank Data (2021) as reported by knoeman.com,, the Nigerian birth rate $\propto_1$ is put at 36.86 per 1000 people per year while the death rate, $\propto_2$, is estimated at 11.4 per 1000 people per year.

The World Health Organisation, WHO (2024) estimated that there are 1 billion annual cases of influenza infection globally. This is approximately 12.28% of the world population of 8.142 billion people as reported by United Nations Population Division (2024). Because there are no specific data on Nigerian influenza awareness rate, we adopt as θ, the figure of Owhonda et al. (2021) in which 39% of the population are reported to be well informed about infection preventive measures. This is because influenza and similar infections such as Covid-19 require the similar forms of information for prevention.

$$\beta_i = 0.39(0.1228) = 0.000528 \text{ per year or } 0.000010163 \text{ per week}$$

$$\beta_u = 0.61(0.1228) = 0.075 \text{ per year or } 0.00144 \text{ per week}$$

The World Health Organisation (2024) also reported that influenza related deaths account for between 250 million to 500 million annual deaths globally. Using the upper limit of 500 million deaths as a fraction the world population, we take the death rate as $\mu = 0.0614$N per year or 0.0011 of the Nigerian population per week

Initial Values

We assume that, the study begins with zero number of treated people. We also assume the initial number of deaths is zero. That is, R(0) = 0, where

$S_i(0)$ = Number of the population members that were informed in the first week

= 0.39 x 213000000 /52= 1,597,500 individuals

Similarly,

$S_u(0)$ = 0.61 x 213,000,000/52= 2,498,654

I(0) = 124800/52 = 2400 individuals

$y_1 = Y_1(0) = \frac{S_1(0)}{N} = \frac{1597500}{2.13\ x10^8} = 0.0075$

$y_2 = Y_2(0) = \frac{S_2(0)}{N} = \frac{2498653.85}{2.13\ x10^8} = 0.0117$

Simulation of Results

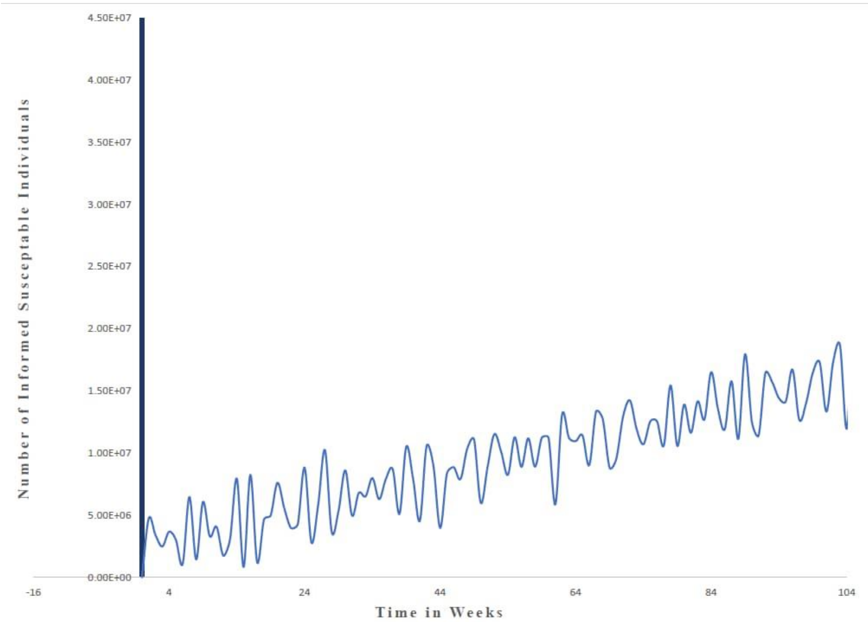
From the data above, we simulated the results in equations (12) and (13) using Gaussian random number generator for the increments $\Delta W_i = W_{(i+1)h} - W_{ih}, i = 0, 1, 2 \ldots, n-1$ of a standard Wiener process. The graph of each disease state is then plotted against time (in weeks) with Microsoft Excel. Each of them is then interpreted from the resulting graph as follows.

Simulation of the Informed Susceptible Class

From the solution $S_i(t)$ we plot the number of informed susceptible individuals, S_i, against time t (Fig.2) where 2 units on the time axis represents one week and one unit on the vertical axis represents 5,000,000 people. The birth rate of 36.855 per 1000people per year is higher than the death rate of 11.4 per1000 people per year. Also, there is transition of Uninformed members to the Informed Class due to creation of awareness, Hence, we notice from fig. 2 that the number of informed susceptible members generally increases from week 1 to week 52. The stochastic effect observed in the graph is due to interaction between the informed class and its 2 immediate classes of Uninformed Susceptible persons and the Infective class.

In other words, $S_i(t)$ rises as the net number of births increases against deaths and more people are informed. On the other hand, $S_i(t)$ drops as informed susceptible persons become infected and are moved to the infective class. Thus, $S_i(t)$ is an increasing function due to high birth rate, despite the transition of its members to infectious the class.

Fig. 2 The Sample Path of Informed Susceptible Individuals Against Time



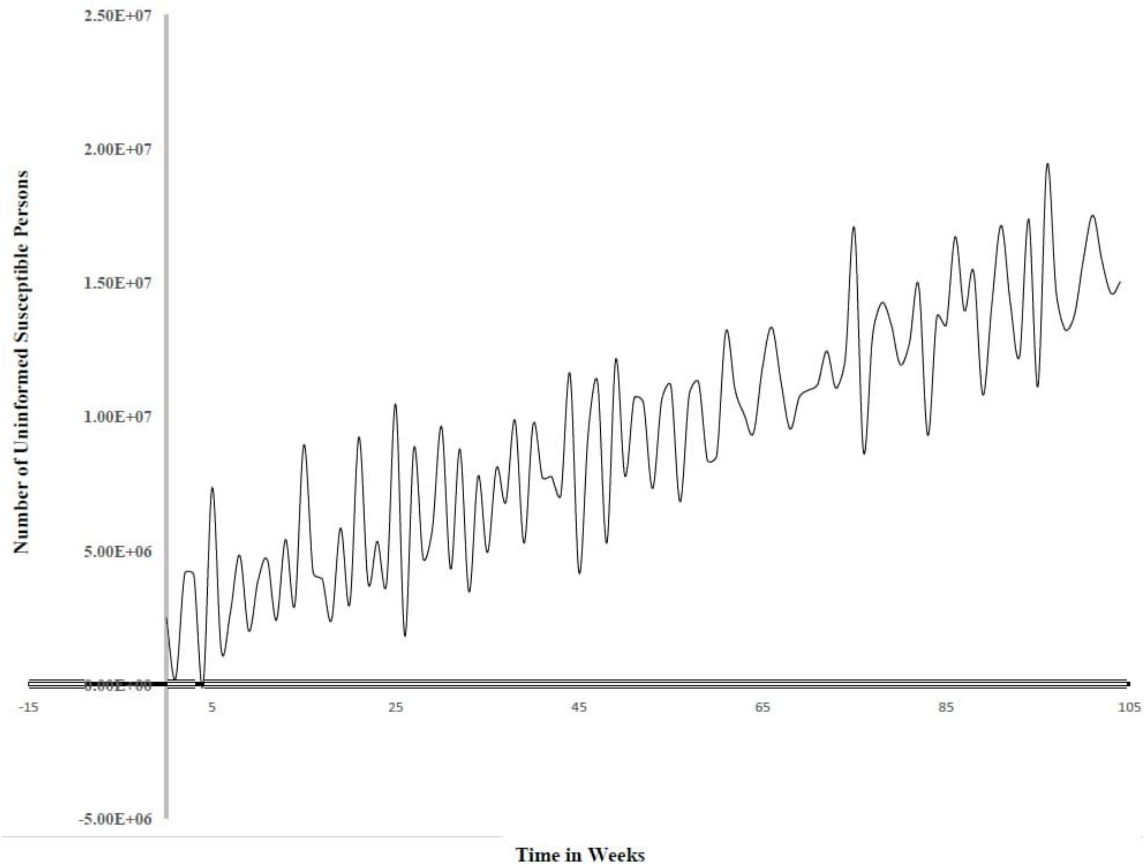
Simulation of the Uninformed Susceptible Class

Using random number generator we simulate the graph of $S_u(t)$ against time using a scale of 2 units to represent 1 week on the horizontal axis and 1 unit to represent 5 x 10^6 uninformed susceptible people on the vertical axis with $S_2(0)$ = 2,498,653 (Fig. 3).

Taking t = 0 as January 1 and t = 104 as the 52^{nd} week of the same year we observe that, like its $S_1(t)$ counterpart, the number of informed people generally rises through the year but with a higher gradient. This rise is partly due to the low rate of information dissemination ($\boldsymbol{\theta = 0.39}$) and partly due to higher birth rate relative to the death rate. We also notice that the graph of $S_u(t)$ oscillates around an increasing mean trajectory. This oscillation is a stochastic effect arising from increase due to birth and successive decrease either due death, movement of its member to Informed class or members' infection and their subsequent transition to the infective class.

In all, the graph depicts a growth in the number of influenza-wise uninformed susceptible members of the Nigerian society, a trend that can be reversed by reduction in birth rate, an aggressive anti-influenza campaign and mass enlightenment programme on preventive measures.

Fig.3: The Sample Path of Uninformed Susceptible Against Time



III. Conclusion

From the analysis of sample paths of the solutions $S_i(t)$ and $S_u(t)$, it is noticed that, despite awareness programme and transition of some individuals to infective class, the sizes of the two susceptible classes keep increasing from week 1 to the 52^{nd} week of the year as an effect of the higher birth rate relative to death rate. This shows that high birth rate is a major factor contributing to increasing spread of influenza in Nigeria. Therefore, birth control programmes should always go alongside other influenza preventive measures such as awareness and vaccination campaign.

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