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Insertive FPT In Ambiguous Situations Under Self-Contractive Mappings With Irrational Conditions

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Abstract
In this study we establish both common and unique fixed point theorems in fuzzy metric spaces under simulation functions, irrational conditions, and self contraction mappings. Here, we get a conclusion on the existence of fixed points by defining a mapping utilising some existing results.
Keywords: *Contraction mapping, Simulation function, Unique fixed point, Complete fuzzy metric space, Banach algebra.*

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I. Introduction

Zadeh1's 1965 invention of fuzzy set theory enables us to describe and deal with ambiguous concepts. Fuzzy sets are essentially functions with a domain that is a non-empty set and a range that is the interval [0,1]. Initially, Weiss2 and Butnariu3 investigated fuzzy mappings' fixed points. Heilpern4 developed the idea of fuzzy contraction mappings (see also 5, 6). Following that, a number of authors determined the existence of fixed points of mappings involving specific contractive type conditions. For instance, Kanwal et al.7 obtained fuzzy common fixed points of fuzzy mappings for integral type contractions, while Azam et al.8,9 established fuzzy fixed points and common fixed points. Additionally, Kanwal et al. generated fuzzy fixed point results utilising contractions of Nadler's type (10,11). A weaker condition was applied in this metric space in place of the triangle inequality in order to generalise the Banach contraction principle.

Definition Let T be the family of all functions $T: R_+ \to R$ such that
(a) T is strictly increasing $i.e. \forall\, x, y \in R_+$ such that $x < y, T(x) < T(y)$
(b) for each sequence $\{a_n\}_{n=1}^{\infty}$ of positive numbers, $\lim_{n\to\infty} T\{a_n\} = -\infty$
(c) there exists $\tau \in (0,1)$ such that $\lim_{n\to\infty} a^k T(a) = 0$.
Definition Let (X, T) be a metric space. A mapping $T_1: X \to X$ is stated to be an τ- contraction on (X, T) if there exist $\tau \in \boldsymbol{T}$ and $\epsilon > 0$ such that, for all $x, y \in X, \Rightarrow\ \epsilon + \tau\big(T(T_1x, T_1y)\big) \le \tau\big(T(x, y)\big)$.
Theorem Let (X, T) be a complete metric space. A mapping $T_1: X \to X$ is stated to be a contraction on (X, T). Then on T has unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.
Definition Let (X, T) be a metric space. A mapping $T_1: X \to X$ is stated to be an τ- weak contraction on (X, T) if there exist $\tau \in \boldsymbol{T}$ and $\epsilon > 0$ such that, for all $x, y \in X$, $T(T_1x, T_1y) > 0 \Rightarrow\ \epsilon + \tau\big(T(T_1x, T_1y)\big) \le \tau\big(M(x, y)\big)$ in which $M(x, y) = max\left\{T(x, y), T(x, T_1x), T(y, T_1y), \frac{T(x,T_1y)+T(y,T_1x)}{2}\right\}$.
Theorem Let (X, T) be a complete metric space. A mapping $T_1: X \to X$ is stated to be an τ- weak contraction on (X, T). If T_1or τ is continuous, then T_1 has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.
Definition Let (X, T) be a metric space. A mapping $T_1: X \to X$ is stated to be a generalized τ- contraction on (X, T) if there exist $\tau \in \boldsymbol{T}$and $\epsilon > 0$ such that, for all $x, y \in X$, $T(T_1x, T_1y) > 0 \Rightarrow\ \epsilon + \tau\big(T(T_1x, T_1y)\big) \le \tau\big(N(x, y)\big)$ in which

$$N(x, y) = max\left\{T(x, y), T(x, T_1x), T(y, T_1y), \frac{T(x, T_1y) + T(y, T_1x)}{2}, \frac{T(T_1^2x,\ x) + T(T_1^2x, T_1y)}{2}, T(T_1^2x, T_1x), T(T_1^2x, y)\, T(T_1^2x, T_1y)\right\}$$

Theorem Let (X, T) be a complete metric space. A mapping $T_1: X \to X$ is stated to be a generalized τ-contraction mapping on (X, T). If T_1or τ is continuous, then T_1 has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Theorem Let T be a self mapping of a complete metric space X into itself. Suppose that there exists $\tau \in \boldsymbol{T}$ and $\epsilon > 0$ such that, for all $x, y \in X$, $T(T_1x, T_1y) > 0 \Rightarrow \epsilon + \tau(T(T_1x, T_1y)) \leq \tau(T(x, y))$. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Theorem Let T be a self mapping of a complete metric space X into itself. Suppose that there exists $\tau \in \boldsymbol{T}$ and $\epsilon > 0$ such that, for all $x, y \in X$, $\frac{1}{2}T(x, T_1x) < T(x, T_1x) \Rightarrow \epsilon + \tau(T(T_1x, T_1y)) \leq \tau(T(x, y))$. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

II. Our Results

Theorem 2.1 Let $(I^X, T_I x)$ be a complete metric space over Banach algebra A and let $f: I^X \to I^X$ be a contractive mapping satisfying the following conditions

$$T_I x\left(f(p_x^\alpha), f(p_y^\beta)\right) \leq k\left(T_I x\left(p_x^\alpha, f(p_y^\beta)\right), T_I x\left(f(p_x^\alpha), p_y^\beta\right)\right)$$

Letting, $p_{x_0}^{\alpha_0} \in I^X$ and suppose $p_{x_1}^{\alpha_1} = f(p_{x_0}^{\alpha_0})$, $p_{x_2}^{\alpha_2} = f(p_{x_1}^{\alpha_1}) = f\left(f(p_{x_0}^{\alpha_0})\right) = f^2(p_{x_0}^{\alpha_0})$, $f(p_{x_3}^{\alpha_3}) = f(p_{x_2}^{\alpha_2}) = f\left(f^2(p_{x_0}^{\alpha_0})\right) = f^3(p_{x_0}^{\alpha_0}), \dots, p_{x_n}^{\alpha_n} = f^n(p_{x_0}^{\alpha_0})$.

Now,
$$\begin{aligned}
T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) &= k\left(T_I x\left(p_{x_n}^{\alpha_n}, f(p_{x_{n-1}}^{\alpha_{n-1}})\right) + T_I x\left(f(p_{x_n}^{\alpha_n}), p_{x_{n-1}}^{\alpha_{n-1}}\right)\right) \\
&\leq k\left(T_I x(p_{x_n}^{\alpha_n}, p_{x_n}^{\alpha_n}) + T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_{n-1}}^{\alpha_{n-1}})\right) \\
&\leq k\left(T_I x(p_{x_n}^{\alpha_n}, p_{x_n}^{\alpha_n}) + T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) - T_I x_{p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}} + T_I x(p_{x_n}^{\alpha_n}, p_{x_{n-1}}^{\alpha_{n-1}}) - T_I x_{p_{x_n}^{\alpha_n}, p_{x_{n-1}}^{\alpha_{n-1}}} + \right. \\
&\qquad \left. + T_I x_{p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_{n-1}}^{\alpha_{n-1}}}\right) \\
&= k\big(T_I x(p_{x_n}^{\alpha_n}, p_{x_n}^{\alpha_n}) + T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) - T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_{n+1}}^{\alpha_{n+1}}) + T_I x(p_{x_n}^{\alpha_n}, p_{x_{n-1}}^{\alpha_{n-1}}) \\
&\qquad - T_I x(p_{x_n}^{\alpha_n}, p_{x_n}^{\alpha_n}) + T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_{n-1}}^{\alpha_{n-1}})\big) \\
&= k\left(T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) + T_I x(p_{x_n}^{\alpha_n}, p_{x_{n-1}}^{\alpha_{n-1}})\right) \\
(1-k) T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) &\leq k T_I x(p_{x_n}^{\alpha_n}, p_{x_{n-1}}^{\alpha_{n-1}})
\end{aligned}$$

$T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) \leq h T_I x(p_{x_n}^{\alpha_n}, p_{x_{n-1}}^{\alpha_{n-1}})$ where $h = k(1-k)^{-1}$

Continuing in the same argument, we will get

$$T_I x(p_{x_{n+1}}^{\alpha_{n+1}}, p_{x_n}^{\alpha_n}) \leq h^{n+1} T_I x(p_{x_0}^{\alpha_0}, p_{x_1}^{\alpha_1})$$

Ofcourse, $\frac{\|T_I x(p_{x_0}^{\alpha_0}, p_{x_1}^{\alpha_1})\| \|h^{n+1}\|}{\|h^{n+1} T_I x(p_{x_0}^{\alpha_0}, p_{x_1}^{\alpha_1})\|} \geq 1$ and its approaches to zero as n approaches to ∞ and hence $\{p_{x_n}^{\alpha_n}\}$ is a Cauchy sequence in I^X. So, by completeness of I^X, there exist $p_x^\alpha \in I^X$ such that $p_{x_n}^{\alpha_n} \to p_x^\alpha$ is a fixed of f. Since, f is continuous, then $f(p_{x_n}^{\alpha_n}) \to f(p_x^\alpha)$, $i.e. p_{x_{n+1}}^{\alpha_{n+1}} \to f(p_x^\alpha)$ and so p_x^α is a fixed point of this mapping. Now, in order to show the uniqueness assume that y is an another fixed point of f. So,

$$\begin{aligned}
T_I x(p_x^\alpha, p_y^\beta) &= T_I x\left(f(p_x^\alpha), f(p_y^\beta)\right) \\
&\leq \lambda\left(T_I x\left(p_x^\alpha, f(p_y^\beta)\right) + T_I x\left(p_y^\beta, f(p_y^\beta)\right)\right) \\
&= \lambda\left(T_I x(p_x^\alpha, p_x^\alpha) + T_I x(p_y^\beta, p_y^\beta)\right)
\end{aligned}$$

Thus, $T_I x(p_x^\alpha, p_y^\beta) = \theta$ and hence $p_x^\alpha = p_y^\beta$. Therefore,p_x^α is a unique fixed point of the mapping f. This finishes the proof.

Theorem 2.2 Let $(I^X, \leq)$be a Semi Ordered Fuzzy Set (SOFS) and $(I^X, T_I x)$ complete fuzzy metric space. If there exist a simulation function τ and $T_1, T_2: I^X \to I^X$ such that

$\tau\left(T_I x\left(T_1(p_x^\alpha), T_1(p_y^\beta)\right), T_I x\left(T_2(p_x^\alpha), T_2(p_y^\beta)\right)\right) \geq 0 \ \forall \ p_x^\alpha, p_y^\beta \in I^X: T_2(p_x^\alpha) \leq T_2(p_y^\beta)$,

Satisfies the following conditions

(a) $T_1 I^X \subseteq T_2 I^X$ and $T_2 I^X$ is closed,

(b) T_1 is T_2 −nondecreasing,

(c) If $\{T_2 p_{x_n}^{\alpha_n}\} \subset I^X$ is a non decreasing sequence, which converges to $T_2 p_x^\alpha$ in $T_2 I^X$, then $T_2 p_{x_n}^{\alpha_n} \leq T_2 p_x^\alpha$ for all $n \geq 0$.

If there exists $p_{x_0}^{\alpha_0} \in I^X$ such that $T_2 p_{x_0}^{\alpha_0} \leq T_1 p_{x_0}^{\alpha_0}$, then T_1 and T_2 have a coincidence point, that is, there exists $p_y^\beta \in I^X$ such that $T_1 p_y^\beta = T_2 p_y^\beta$.

Proof. Using the theorem condition, we have $p_{x_0}^{\alpha_0} \in I^X$ such that $T_2 p_{x_0}^{\alpha_0} \leq T_1 p_{x_0}^{\alpha_0}$. Since $T_1 I^X \subseteq T_2 I^X$, then there exists $p_{x_1}^{\alpha_1} \in I^X$such that $T_2 p_{x_1}^{\alpha_1} \leq T_1 p_{x_0}^{\alpha_0}$ and $T_2 p_{x_0}^{\alpha_0} \leq T_1 p_{x_0}^{\alpha_0} = T_2 p_{x_1}^{\alpha_1}$. Since T_1 is T_2 −nondecreasing, we have $T_1 p_{x_0}^{\alpha_0} \leq T_1 p_{x_1}^{\alpha_1}$. Continuing this process, we construct the sequence $\{p_{x_n}^{\alpha_n}\}$ with the following conditions
$T_1 p_{x_{n-1}}^{\alpha_{n-1}} = T_2 p_{x_n}^{\alpha_n}$ for all $n \geq 0$,
and $T_2 p_{x_0}^{\alpha_0} \leq T_1 p_{x_0}^{\alpha_0} = T_2 p_{x_1}^{\alpha_1} \leq T_1 p_{x_1}^{\alpha_1} = T_2 p_{x_2}^{\alpha_2} \leq T_1 p_{x_2}^{\alpha_2} \leq \cdots \leq T_1 p_{x_{n-1}}^{\alpha_{n-1}} = T_2 p_{x_n}^{\alpha_n} \leq T_1 p_{x_n}^{\alpha_n} = T_2 p_{x_{n+1}}^{\alpha_{n+1}} \leq \cdots$
If two consecutive members of the sequence $\{T_1 p_{x_n}^{\alpha_n}\}$ or $\{T_2 p_{x_n}^{\alpha_n}\}$ are equal, then the conclusion of the theorem follows. So we have $T_I x(T_1 p_{x_n}^{\alpha_n}, T_1 p_{x_{n+1}}^{\alpha_{n+1}}) \neq 0$, $T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_{x_{n+1}}^{\alpha_{n+1}}) \neq 0$ for all $n \geq 0$.
If for some $n \in N$, we assume that $T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n}) < T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_{x_{n+1}}^{\alpha_{n+1}})$, then by property of simulation function and (2)-(4) we have

$$0 \leq \tau\left(T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n}), T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n})\right)$$
$$= \tau\left(T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n}), T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n})\right)$$
$$< T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n}) - T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_{x_{n+1}}^{\alpha_{n+1}}) < 0.$$

This contradiction shows that
$T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_{x_{n+1}}^{\alpha_{n+1}}) \leq T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n})$.
This implies that the sequence $\{T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n})\}$ is a monotonic decreasing sequence of non negative real numbers, and consequently, there exists $r \geq 0$ such that the sequence $\{T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n})\}$ converges to r.
Suppose $r > 0$. By (3) we know that the elements $T_2 p_{x_n}^{\alpha_n}$ and $T_2 p_{x_{n+1}}^{\alpha_{n+1}}$ are comparable, so using the property of a simulation function with $\theta_n = T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_{x_{n+1}}^{\alpha_{n+1}})$ and $\emptyset_n = (T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n})$, we have

$$0 \leq \lim_{n\to\infty} \sup \tau\left(T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n}), T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n})\right)$$
$$= \lim_{n\to\infty} \sup \tau\left(T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n}), T_I x(T_1 p_{x_{n-1}}^{\alpha_{n-1}}, T_1 p_{x_n}^{\alpha_n})\right) < 0,$$

which is a contradiction, and hence,

$$\lim_{n\to\infty} T_I x(T_2 p_{x_{n-1}}^{\alpha_{n-1}}, T_2 p_{x_n}^{\alpha_n}) = 0$$

Now, we shall show that the sequence $\{T_2 p_{x_n}^{\alpha_n}\}$ is Cauchy. By contradiction there exists $\epsilon > 0$ and $\left\{T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}\right\}, \left\{T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}\right\} \subset \{T_2 p_{x_n}^{\alpha_n}\}$ with $n(k) > m(k) \geq k \;\forall\; k \in N$ such that
$\lim_{k\to\infty} T_I x\left(T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}, T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}\right) = \lim_{k\to\infty} T_I x\left(T_2 p_{x_{m(k)+1}}^{\alpha_{m(k)+1}}, T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}\right) = \epsilon$,
$T_I x\left(T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}, T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}\right) \geq \epsilon$.
Then we can assume that $T_I x\left(T_2 p_{x_{m(k)+1}}^{\alpha_{m(k)+1}}, T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}\right) > 0 \;\forall k \in N$. Again, by (3) we know that the elements $T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}$ and $T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}$ are comparable, so using (5)-(7) and the property of a simulation function $\theta_n = T_I x\left(T_2 p_{x_{m(k)+1}}^{\alpha_{m(k)+1}}, T_2 p_{x_{n(k)+1}}^{\alpha_{n(k)+1}}\right)$and $\emptyset_n = T_I x\left(T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}, T_2 p_{x_{n(k)}}^{\alpha_{n(k)}}\right)$, we have

$$0 \leq \lim_{k\to\infty} \sup \tau\left(T_I x\left(T_1 p_{x_{m(k)}}^{\alpha_{m(k)}}, T_1 p_{x_{n(k)}}^{\alpha_{n\,(k)}}\right), T_I x\left(T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}, T_2 p_{x_{n(k)}}^{\alpha_{n\,(k)}}\right)\right)$$
$$= \lim_{k\to\infty} \sup \tau\left(T_I x\left(T_2 p_{x_{m(k)+1}}^{\alpha_{m(k)+1}}, T_2 p_{x_{n(k)+1}}^{\alpha_{n\,(k)+1}}\right), T_I x\left(T_2 p_{x_{m(k)}}^{\alpha_{m(k)}}, T_2 p_{x_{n(k)}}^{\alpha_{n\,(k)}}\right)\right) < 0$$

Which is a contradiction. We conclude that the sequence $\{T_2 p_{x_n}^{\alpha_n}\}$ is a Cauchy sequence, and hence, $\{T_2 p_{x_n}^{\alpha_n}\}$ is convergent in the complete fuzzy metric space $(I^X, T_I x)$. $T_2 I^X$ is closed, therefore by (2) there exists $p_x^{\alpha} \in I^X$ such that $\lim_{n\to\infty} T_1 p_{x_n}^{\alpha_n} = \lim_{n\to\infty} T_2 p_{x_n}^{\alpha_n} = T_2 p_x^{\alpha}$.
From (3) and (8) we know that $\{T_2 p_{x_n}^{\alpha_n}\}$ is a non decreasing sequence in $T_2 I^X$ such that $T_2 p_{x_n}^{\alpha_n} \to T_2 p_x^{\alpha}$, then by condition (c) and (4) we have $T_2 p_{x_n}^{\alpha_n} \leq T_2 p_x^{\alpha}$. Again, by (3) and (4) since T_1isT_2-non decreasing, we have $T_1 p_{x_n}^{\alpha_n} \leq T_1 p_x^{\alpha}$. Now using the property of a simulation function, (9) and (10), we have

$$0 \leq \tau\left(T_I x(T_1 p_{x_n}^{\alpha_n}, T_1 p_x^{\alpha}), T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_x^{\alpha})\right) < T_I x(T_2 p_{x_n}^{\alpha_n}, T_2 p_x^{\alpha}) - T_I x(T_1 p_{x_n}^{\alpha_n}, T_1 p_x^{\alpha})$$

$\forall\; n \in N$.
Taking $n \to \infty$ in the above inequality, we have $\lim_{n\to\infty} T_1 p_{x_n}^{\alpha_n} = T_1 p_x^{\alpha}$. Then
$T_1 p_x^{\alpha} = \lim_{n\to\infty} T_1 p_{x_n}^{\alpha_n} = \lim_{n\to\infty} T_2 p_{x_n}^{\alpha_n} = T_2 p_x^{\alpha}$.
Now, in order to show the uniqueness, suppose that the set of fixed points of T_2 is totally ordered. Assume on the contrary that $p_x^{\alpha} = T_1 p_x^{\alpha} = T_2 p_x^{\alpha}$ and $p_y^{\beta} = T_1 p_y^{\beta} = T_2 p_y^{\beta}$ but$x \neq y$ .Sincep_x^{α} andp_y^{β} contain a set of fixed points ofT_2 without loss of generality, we assume that$T_1 p_x^{\alpha} \leq T_2 p_y^{\beta}$. If$T_1 p_x^{\alpha} \leq T_1 p_y^{\beta}$ or $T_2 p_x^{\alpha} \leq T_2 p_y^{\beta}$. Then$p_x^{\alpha} =$

p_y^β,which is a contradiction, otherwise$T_1 p_x^\alpha \neq T_1 p_y^\beta$ and$T_2 p_x^\alpha \neq T_2 p_y^\beta$. Now, by the property of simulation we have

$$0 \leq \tau\left(T\left(T_1 p_x^\alpha, T_1 p_y^\beta\right), T\left(T_2 p_x^\alpha, T_1 p_y^\beta\right)\right)$$
$$= \tau\left(T\left(p_x^\alpha, p_y^\beta\right), T\left(p_x^\alpha, p_y^\beta\right)\right)$$

$< \tau\left(T\left(p_x^\alpha, p_y^\beta\right), T\left(p_x^\alpha, p_y^\beta\right)\right) = 0,$

Which is a contradiction. Therefore, T_1andT_2have a unique common fixed point. Hence the result.

III. Conclusion

Insertive fixed-point theorems, often framed within the context of weakly compatible or coincidence point theory, have been successfully adapted to fuzzy metric spaces to study the behavior of self-contractive mappings. These theorems extend traditional metric space findings to fuzzy settings.

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