

Application of Linear Programming Technique in the Determination of Optimum Production Capacity

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Abstract: *In this paper, we were concerned with applying linear programming technique to determine optimum production of Usmer Water Company, Uyo. TORA Software was used in the analysis of the data using M – method. The result showed the values of the decision variables, x_1, x_2, x_3, x_4 and x_5 to be 95, 0, 5.9, 10 and 17 respectively. Sensitivity Analysis of the problem was also discussed.*

Keywords: *Linear Programming, Optimum Production, Model, Maximization.*

I. Introduction:

Optimization has become a common phenomenon in almost all organizations and establishments. In developed, countries managerial decisions are mostly based on the use of optimization techniques. In a profit seeking organizations, the Board of Directors has many things to tackle which may include: The problem of other competitors in the same business, availability of funds for new capital projects, reduction of operational cost, high level of output and ultimately maximization of profit as explained by Nonso (2005).

In an attempt to address these problems there are two techniques of operation that may be applied. They include: The quantitative technique and the qualitative technique. Quantitative technique which is preferred involves modeling of a ‘real form’ problem into a mathematical form which can be solved to arrive at a Solution that would aid the decision makers. Linear programming (LP) technique is such a Quantitative technique: It is a widely used Mathematical modeling technique concerned with the efficient allocation of limited resources to known activities with the objective of meeting the desired goal (Taha, 1977).

In 1947, during World War II, George B. Dantzing while working with the US Air force, developed LP model primarily for solving military logistics problems. But now, it is extensively being used in all functional areas of management, airlines, agriculture, military operations, oil refining, education, energy planning, health care system etc.

Frizzone, et al (1997) used linear programming model to optimize the water resource used in irrigation, and obtained an optimal way of water resource usage. Though, considering the numerous constraints, he had to develop a separate mathematical model to achieve his goal. He noted that the mathematical programming quantifies an optimal way of combing scarce resources to satisfy the proposed goals; that is to analyze the cases where the available resources must be combined in a way to maximize the profit or minimize cost.

Nonso (2005) in his work on Application of Linear programming for Managerial Decision found out how an organization can have effective control over materials for input during production. He observed that units produced must be assumed as what is sold in order to achieve the company’s goal.

Fagoyinbo, I. S. and Ajibode, I. A. (2010) worked on the Application of Linear programming Techniques in the Effective use of resources for staff training. The method employed gave an integer optimum solution to all the models formulated. The Data used did not yield a feasible solution but when the model reformed gave an optimum solution.

B. I. Ezema and U. Amakom (2012) worked on optimizing profit with the linear programming model: A focus on Golden plastic Industry limited. Enugu. 2012. The result they had showed that only 2 sizes of the total 8 ‘PVC’ pipes should be produced.

I. U.Khan, N. H. Bajuri and I. A. Jadoon (2011) in their work, optimal production levels for the different products manufactured at ICL, a multinational Company in Pakistan had a result that showed that the amount was raised by changing production patterns within the first, second, third and fourth digit respectively. The intention of this paper is therefore to determine the optimum production capacity of Usmer water company, Uyo, Nigeria.

II. Methodology

Linear Programming is a mathematical technique for generating and selecting the optimal or the best solution for a given objective function. It may be defined as a method of optimizing (i.e maximizing or minimizing) a linear function for a number of constraints stated in the form of linear equations.

The general linear programming problem (or model) with n decision variables and m constraints can be stated in the following form.

(Max or min) $Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$
 Subject to Linear Constraints of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq, =, \geq) b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq, =, \geq) b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq, =, \geq) b_m \\ x_1 + x_2 + \dots + x_n \geq 0 \end{aligned}$$

The above formulation can also be expressed in a compact form. Using summation sign (Max or Min) $Z = \sum_{j=1}^n c_j x_j$ (Objective function) Subject to the linear constraints $\sum_{j=1}^n a_{ij} x_j (\leq, =, \geq) b_i, i = 1, 2, \dots, m$... (Constraints) and $x_j \geq 0, j = 1, 2, \dots, n$ (Non-negativity conditions). Specifically the linear programming model for this paper is given by

Maximize $Z = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5$ (objective function)
 Subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 &\leq b_1 \text{ (Cost constraint)} \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 &\leq b_2 \text{ (Production time Constraint)} \\ a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + a_{i4}x_4 + a_{i5}x_5 &\leq b_i \text{ (Demand or sales constraint)} \\ x_1, x_2, x_3, x_4, x_5 &\geq 0. \end{aligned}$$

And the assumptions for this model are

Model Assumptions

- a. The production quantity is the same as the sales. As it is unrealistic to produce what is not sold.
- b. There is a linear relationship among the variables used in the model

Where x_1 = Sachet Bag of water
 x_2 = 50cl (carton) of water
 x_3 = 75cl (carton) of water
 x_4 = 1.5 liters (carton) of water
 x_5 = 10 liters Refillable of water

And c_k = Product Price – Product Cost
 $k = 1, 2, \dots, 5$ (ie $c_1 = 45, c_2 = 200, c_3 = 250, c_4 = 450, c_5 = 500$)

III. Data Presentation

The data for this study are in the table 1 below which were extracted from the sales record of Usmer water company (Uyo, Nigeria) in September, 2012.

Table 1

S/n	Product Package	Production Time(mins)	Production Cost (N)	Selling Price (N)	Lowest Purchase (Daily)	Highest Purchase (Daily)
1	Sachet Bag	3.3	35	80	95	883 bags
2	50cl (carton)	6.8	200	400	0	10 cartons
3	75cl (carton)	7.2	250	500	0	32 cartons
4	1.5 liters (carton)	9.0	400	950	0	10 cartons
5	10 liters Refillable	2.0	0	500	1 drum	178 drum

Working Hours 8am – 4Pm daily ie 480mins (8 hrs)

The Formulated Model is given as
 Maximize $45x_1 + 200x_2 + 250x_3 + 450x_4 + 500x_5$
 Subject to.
 $35x_1 + 200x_2 + 250x_3 + 400x_4 + 0x_5 \leq 20000$
 $3.3x_1 + 6.8x_2 + 7.2x_3 + 9.0x_4 + 2x_5 \leq 480$
 $x_1 \leq 883$

$$\begin{array}{rcl}
 x_2 & & \leq 10 \\
 & x_3 & \leq 32 \\
 & & x_4 \leq 10 \\
 x_1 & & x_5 \leq 17 \\
 & & \geq 95 \\
 & & x_5 \geq 1 \\
 x_1, x_2, x_3, x_4, x_5 & \geq & 0
 \end{array}$$

IV. Data Analysis and Results

The data were analyzed using computer software – TORA and the solution is given thus $x_1 = 45, x_2 = 0, x_3 = 5.9, x_4 = 10, x_5 = 17$.

Therefore the final solution is

$$\text{Maximize } Z = 45(95) + 200(0) + 250(5.9) + 450(10) + 500(17) = 18,750$$

The summary of the result is given in Table 2 below

Table 2

S/n	Variable	Value	Obj coeff	Obj val.contribution
1	Sachet	95	45	4275
2	50 cl	0	200	0
3	75 cl	5.9	250	1475
4	1.5 liters	10	450	450
5	10 liters	17	500	8500

Constraint	RHS	Black -/Surplant
<	20000	9199.3/-
<	480	0
<	883	788-
<	10	10-
<	32	26.10-
<	10	0
<	17	0
>	95	0
>	1	0

V. Discussion

The results or solutions in table 2 require that the company shows produce daily,

- 95 Bags of Sachet water
- 0 Cartons of 50cl water
- 65 Cartons of 75cl water
- 10 Cartons of 1.5 liters water
- 17 drums of refillable 10liters

Note that a bag contains 20 Sachets while a carton contains 12 bottles. Also it is noticed that if the company embarks on this plan the following results would follow|:

- a. ~~₦~~9199.31 of ₦20,000 (Maximum Production Cost) shall be unused cost.
- b. Total working time of 480 minutes shall be fully exhausted /utilized.
- c. A deficit production of 788 bags of sachet water may occur if demand increased to 883 bags and would require
- d. 26 bottles of 75cl bottles shall be deficit production.
- e. Considering the sales records the company should be able to meet up with demand for 1.5 liters bottled and 10 liters refillable drum of water

VI. Sensitivity Analysis

This was performed still using Tora software and the result is as follows in Table 3.

Table 3

S/n	Variable	Current Obj coefficient	Min Obj Coefficient	Max obj Coefficient	D Price
1	Sachet bag	45	-infinity	114.58	0
2	50cl	200	-infinity	236.11	36.11
3	75cl	250	211.76	360	0
4	1.5 liters	450	312.50	infinity	0
5	10 refillable	500	69.44	infinity	0

S/n	Constraints	Current	Min RHs	Max RHs	D Price
1	<	20,000	10800.69	Infinity	0
2	<	480	437.50	667.9	34.72
3	<	883	95	Infinity	0
4	<	10	0	Infinity	0
5	<	32	5.9	Infinity	0
6	<	10	0	14.72	137.5
7	<	17	0	38.25	430.56
8	>	95	38.06	107.88	69.58
9	>	1	0	5.6	0

Conclusion: linear programming method has proven, given the raw data the only way the company production problem can be addressed.

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