

# Hydro Magnetic Free Convection Mass Transfer Flow of A Polar Fluid on A Vertical Porous Plate with Couple Stress in Slip Flow Regime

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**Abstract:** This paper examines the problem of MHD steady mass transfer flow of a polar fluid through a porous medium bounded by an infinite vertical porous plate in slip flow regime. In the mass transfer equation effect of thermal diffusivity which plays an important role in the flow is also considered. The exact solutions for velocity field, angular velocity field, temperature distributions and concentration field are obtained. The expression for the skin-friction, the rate of heat transfer are also derived. Effects of rotational parameter ( $\alpha$ ), couple stress parameter ( $\beta$ ) and other parameter entered into the problem are examined with the help of graphs. It is found that the velocity of the fluid is increased when the rotational parameter is increased but decreased in case of couple stress parameter.

**Key words:** Magnetic field, Mass transfer, Porous medium, Rotational velocity, Thermal diffusion.

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## I. Introduction

The study of the flow through a porous medium under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as magnetohydrodynamic generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics. Also, free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal etc. magnetohydrodynamics has attracted the attention of a large number of scholars due to its diverse applications.

In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. in engineering, it finds its applications in MHD pumps, MHD bearings etc. Brinkmann [1], Chawla and Singh [2], Yamamoto and Yoshida [3] have discussed fluid flow through porous medium considering generalized Darcy's law which contains the terms of viscous stress and convection acceleration. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices is given in the book of Nield and Bejan [4]. In all above works, generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles.

Aero et al. [5] derived and solved the flow equations for the fluid in which angular velocity of the fluid particles was taken into account. These fluids are known as polar fluids in the literature and are more general than ordinary fluids Lukaszewicz [6]. Rapits et al. [7], Raptis and Takhar [8] have discussed polar flow through porous medium. Recently Saxena and Dubey [9], Jain et al. [10], Jain and Gupta [11, 12] considered magnetic effects on polar flow. Polar fluids belong to a class of fluids with microstructure and have asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. Some authors viz. Ahmadi [13], Kim [14] used the word micropolar in place of polar because of the microstructure property of the fluid particles. From technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetosphere, aeronautics, chemical engineering and electronics on account of their varied importance, these flows have been studied by several authors notable amongst them are Shercliff [15], Ferraro and Plumpton [16] and Cramer [17].

Radiative convective flows are encountered in countless industrial and environment processes, such as heating and cooling chambers, fossil fuel combustion energy processes evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. England and Emery [18] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary plate.

Chandrakala and Antony Raj [19], Soundalgekar and Takhar [20] have considered the radiative free convective flow past a semi-infinite vertical plate. As slip boundary conditions are very useful in geothermal region in which situations may arise when the flow becomes unsteady and slip at the boundary takes place. Recently Jothimani and Anjali Devi [21] and Khandelwal and Jain [22] have considered the slip boundary conditions in their heat flow problems.

In the present study, we can consider electrically conducting steady polar flow through porous medium in slip flow regime under the influence of radiative heat. As whole of the body is not rotating but only randomly oriented fluid particles are rotating. Coriolis force is not considered (French and Norton [23]) in this problem.

Obtaining exact solutions, effects of different parameters entering into the problem namely K (permeability parameter), M (magnetic parameter),  $\alpha$  (kinematic rotational viscosity parameter),  $h_1$  (velocity slip parameter),  $\beta$  (couple stress parameter), Gr (thermal Grashof number), Gc (mass Grashof number), So (soret number), Sc (Schmidt number), S (radiation parameter) and Pr (Prandtl number) are discussed on the velocity field, angular velocity and temperature, concentration, skin friction and quantity of heat transfer and are shown graphically.

## II. Mathematical formulation and solutions of the problem

We consider the two dimensional, steady, free convective flow with mass transfer and thermal diffusion of an electrically conducting polar fluid through a porous medium in slip flow regime past an infinite vertical porous plate in presence of a heat source/sink. A magnetic field of uniform strength is applied transversely to the direction of the flow. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. We taken the x-axis along the plate and y-axis normal to it and the flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the fluid. The suction velocity  $v_0$  and permeability K of the porous medium are constant.

Under these conditions using the Boussinesque's approximation equations, governing the flow are given as :

Continuity equation

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Linear momentum equation

$$v \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + g\beta^1(C - C_\infty) \frac{1}{\rho} (\mu + \mu_r) \frac{\partial^2 u}{\partial y^2} + \frac{2}{\rho} \mu_r \frac{\partial \omega}{\partial y} - \frac{vu}{K} - \frac{\sigma \mu_e B^2 u}{\rho} \tag{2.2}$$

Angular momentum equation

$$v \frac{\partial \omega}{\partial y} = \frac{\gamma}{I} \frac{\partial^2 \omega}{\partial y^2} \tag{2.3}$$

Energy equation

$$v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \tag{2.4}$$

Concentration equation

$$v \frac{\partial c}{\partial y} = D \frac{\partial^2 T}{\partial y^2} + D\ell \frac{\partial^2 T}{\partial y^2} \tag{2.5}$$

Here  $\rho$  denotes the density,  $u$  the velocity along the x-direction and  $v$  the velocity along the y-axis,  $\mu$  the viscosity of the fluid  $\mu_r$  the rotational viscosity,  $B$  the strength of magnetic field,  $\omega$  and  $T$  are mean angular velocity and temperature of fluid particles.  $\sigma$  is fluid conductivity,  $\mu_e$  is magnetic permeability.  $D$  is the coefficient of chemical molecular diffusivity,  $D\ell$  is thermal diffusivity.  $I$  a scalar constant of dimension equal to that of moment of inertia of unit mass and

$$\gamma = C_a + C_d$$

where  $C_a$  and  $C_d$  are coefficient of couple stress viscosities. Remaining symbols have their usual meanings.

The boundary conditions are :

$$u = L_1 \frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad T = T_w, C = C_w \quad \text{at } y = 0$$

$$u \rightarrow 0, \omega \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \tag{2.6}$$

where  $L_1 = \left( \frac{2 - m_1}{m_1} \right) L$ ,  $L$  being mean free path and  $m_1$  the Maxwell's reflection coefficient.

Integration of equation (2.1) for constant suction gives :

$$v = v_0 \tag{2.7}$$

The radiative heat flux  $q_r$  is considered as

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)I_1 \tag{2.8}$$

where  $I_1 = \int_0^\infty K_{\lambda n} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$ ,

Introducing the following non-dimensional quantities

$$u^* = \frac{u}{v_0}, \quad y^* = \frac{yv_0}{\nu}, \quad \omega^* = \frac{\omega\nu}{v_0},$$

$$K^* = \frac{Kv_0^2}{\nu^2}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\alpha = \frac{\nu_r}{\nu} \text{ (Rotational parameter)}, \quad \beta = \frac{I\nu}{\gamma} \text{ (Couple stress parameter)}$$

$$M^2 = \frac{\sigma\mu_e B^2 \nu}{\rho v_0^2} \text{ (Magnetic field parameter),}$$

$$Gr = \frac{\nu g \beta (T_w - T_\infty)}{v_0^3} \text{ (Thermal Grashof number),}$$

$$Gc = \frac{\nu g \beta^1 (C_w - C_\infty)}{v_0^3} \text{ (Mass Grashof number), } Pr = \frac{\mu C_p}{k} \text{ (Prandtl number),}$$

$$S = \frac{4\nu I_1}{\rho C_p v_0^2} \text{ (Radiation parameter), } h_1 = \frac{L_1 v_0}{\nu} \text{ (Velocity slip parameter),}$$

$$Sc = \frac{\nu}{D} \text{ (Schmidt number), } S_0 = \frac{D\ell(T_w - T_\infty)}{\nu(C_w - C_\infty)} \text{ (Soret number)}$$

In view of equations (2.7), (2.8) the equations of motion in non-dimensional form after dropping the asterisks over them, reduced to :

$$(1 + \alpha) \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + 2\alpha \frac{\partial \omega}{\partial y} + Gr\theta + Gc\phi - N^2 u = 0 \tag{2.9}$$

$$\frac{\partial^2 \omega}{\partial y^2} + \beta \frac{\partial \omega}{\partial y} = 0 \tag{2.10}$$

$$\frac{\partial^2 \theta}{\partial y^2} + Pr \frac{\partial \theta}{\partial y} - Pr S \theta = 0 \tag{2.11}$$

$$\frac{\partial^2 \phi}{\partial y^2} + Sc \frac{\partial \phi}{\partial y} - S_0 Sc \frac{\partial^2 \theta}{\partial y^2} = 0 \tag{2.12}$$

With corresponding boundary conditions as

$$u = h_1 \frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad \theta = 1, \quad \phi = 1 \text{ at } y = 0$$

$$u \rightarrow 0, \omega \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \tag{2.13}$$

where  $N^2 = M^2 + \frac{1}{K}$

Solving the equations (2.9) to (2.12), we get the following exact solution after using corresponding boundary conditions (2.13) :

$$u = C_5 e^{-R_1 y} - E_1 C_1 e^{-\beta y} + (E_2 C_2 - E_4 C_4) e^{-R_3 y} + E_3 C_3 e^{-Sc y} \quad (2.14)$$

$$\omega = C_1 e^{-\beta y} \quad (2.15)$$

$$\theta = C_2 e^{-R_3 y} \quad (2.16)$$

$$\phi = C_3 e^{-Sc y} - C_4 e^{-R_3 y} \quad (2.17)$$

### III. Skin friction and Nusselt number

The skin-friction due to velocity is given by

$$\tau_w = (\mu + \mu_r) \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

In non-dimensional form after dropping the asterisk

$$\tau = \frac{\tau_w}{\rho v_0^2} = (1 + \alpha) \{ -R_1 C_5 + \beta E_1 C_1 + (E_4 C_4 - E_2 C_2) R_3 - Sc E_3 C_3 \} \quad (3.1)$$

Another important parameter, the rate of heat transfer is given by

$$Nu = \frac{-L_1}{(T_w - T_\infty)} \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

In non-dimensional form after dropping the asterisk

$$Nu = h_1 C_2 R_3 \quad (3.2)$$

Where

$$C_1 = \frac{(R_1^2 C_5 + K_1)}{\beta(1 + \beta E_1)}, \quad C_2 = 1, \quad C_3 = 1 + C_4$$

$$C_4 = \frac{S_0 Sc R_3 R_2}{(R_3 - Sc)}, \quad C_5 = \frac{E_1(1 + \beta h_1) K_1 - \beta(1 + \beta E_1) K_2}{\beta(1 + \beta E_1)(1 + h_1 R_1) - R_1^2 E_1(1 + \beta h_1)}$$

$$K_1 = R_2^3 E_2 + Sc^2 E_3 C_3 + R_2^3 E_4 C_4,$$

$$K_2 = R_3 E_2 h_1 + E_3 h_1 Sc C_3 - R_3 E_4 h_1 C_4 + E_2 + E_3 C_3 - E_4 C_4$$

$$R_1 = \frac{1 + \sqrt{1 + 4(1 + \alpha) N^2}}{2(1 + \alpha)},$$

$$R_2 = \frac{-1 + \sqrt{1 + 4(1 + \alpha) N^2}}{2(1 + \alpha)}$$

$$R_3 = \frac{Pr + \sqrt{Pr^2 + 4 Pr S}}{2},$$

$$E_1 = \frac{2\alpha\beta}{(1 + \alpha)(R_1 + \beta)(R_2 - \beta)}$$

$$E_2 = \frac{Gr}{(1 + \alpha)(R_1 + R_3)(R_2 - R_3)},$$

$$E_3 = \frac{Gc}{(1 + \alpha)(R_1 + Sc)(R_2 - Sc)}$$

$$E_4 = \frac{Gc}{(1 + \alpha)(R_1 + R_3)(R_2 - R_3)}$$

IV. Discussions and Conclusions

In order to understand the physical situation, we have calculated the numerical values of the velocity distribution (Figure 1 and Figure 2), Angular velocity distribution (Figure 3 and Figure 4), Temperature distribution (Figure 5), Concentration distribution (Figure 6), skin friction ( Figure 7) Nusselt number (Figure 8) for different parameter entered into the problem

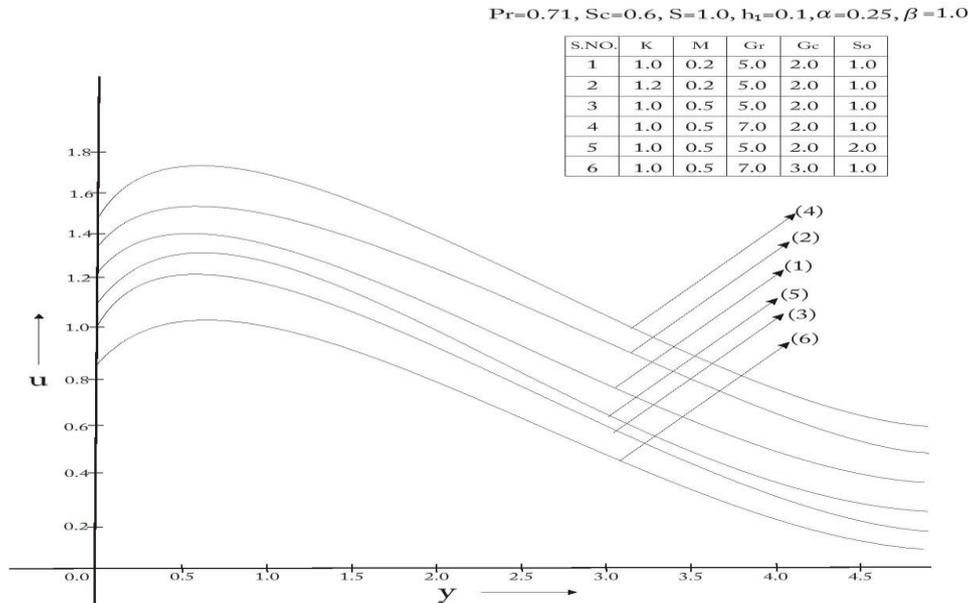


Figure 1. Velocity profile for different values of K, M, Gr, Gc and So

In Figure 1 the velocity distribution is plotted against y for fixed values of  $Pr = 0.71, Sc = 0.6, S = 1.0, h_1 = 0.1, \alpha = 0.25$  and  $\beta = 1.0$ . It is being observed that when K, Gr and  $S_o$  are increased velocity increased but for the case of M and Gc the velocity decreased. Physically, we say that increase in K increased the flow space in the porous medium hence velocity increase.

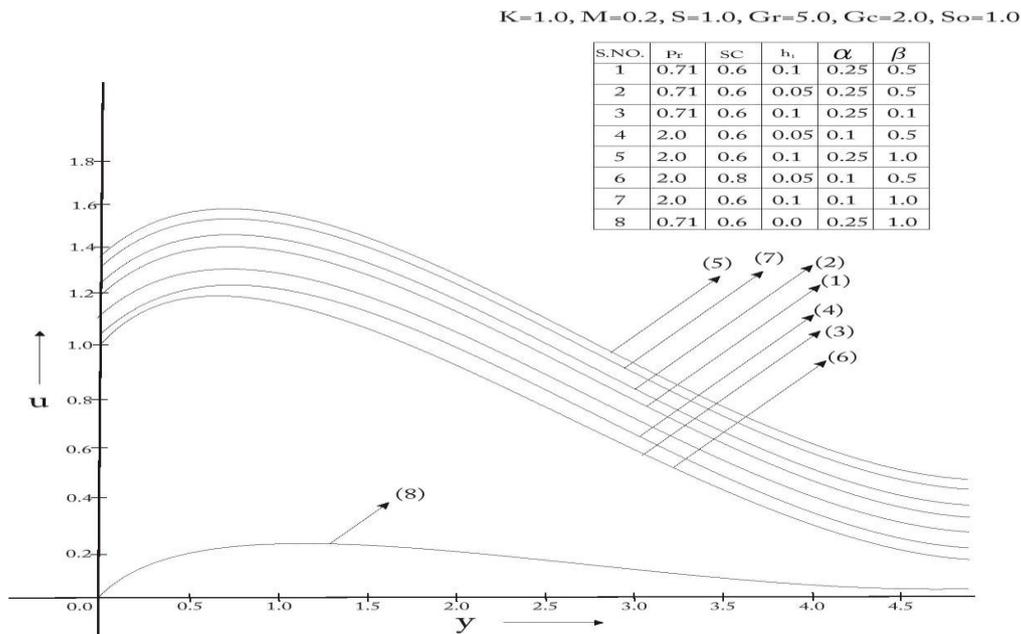


Figure 2. Velocity profile for different values of Pr, Sc,  $h_1, \alpha$  and  $\beta$

In Figure 2, the velocity distribution is plotted against y for  $K = 1.0, M = 0.2, S = 1.0, Gr = 5.0, Gc = 2.0$  and  $S_o = 1.0$ . It is being observed that when Pr,  $h_1$  and  $\alpha$  are increased velocity is increased, but increase in

Sc and  $\beta$  decrease the velocity. It is further observed that when  $h_1 = 0$  (No slip condition) velocity is decreased as compared to when  $h_1 \neq 0$  it means slip velocity increase the velocity of the fluid.

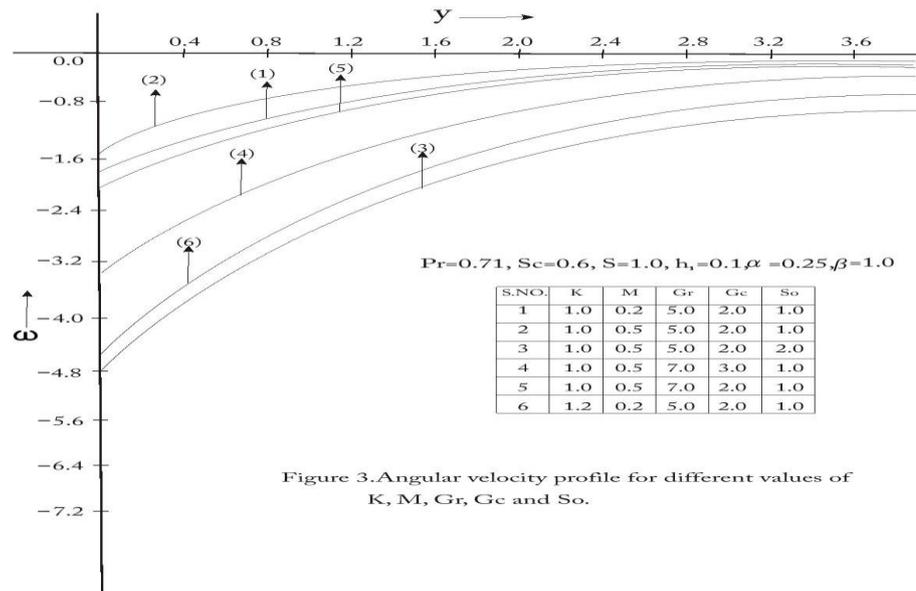


Figure 3. Angular velocity profile for different values of K, M, Gr, Gc and So.

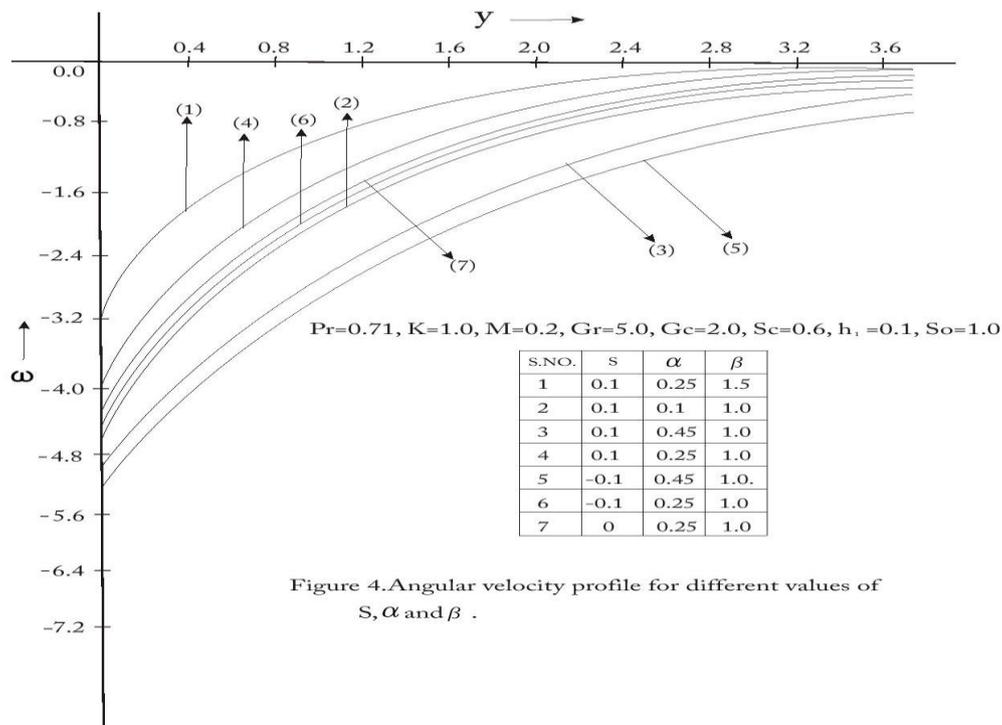


Figure 4. Angular velocity profile for different values of S, α and β .

From figures 3 and 4, we observe that angular velocity is less for higher values of K, Gr, Gc and  $S_0$  and more with the higher values of  $\beta$  and M. As expected, effect of couple stress parameter leads to rise in angular velocity but effect of rotational parameter lead to fall in angular velocity. Moreover, decrease in radiation parameter decreases the angular velocity.

Temperature distribution is plotted against y in Figure 5. It is observed that temperature of the fluid is more in case of air ( $Pr = 0.71$ ) in Comparison with Water ( $Pr = 7.0$ ). Moreover, increase in radiation parameter leads to fall in temperature for both the basic fluids. It is further observed that for the case of water negative of radiation parameter increase the temperature of the fluid.

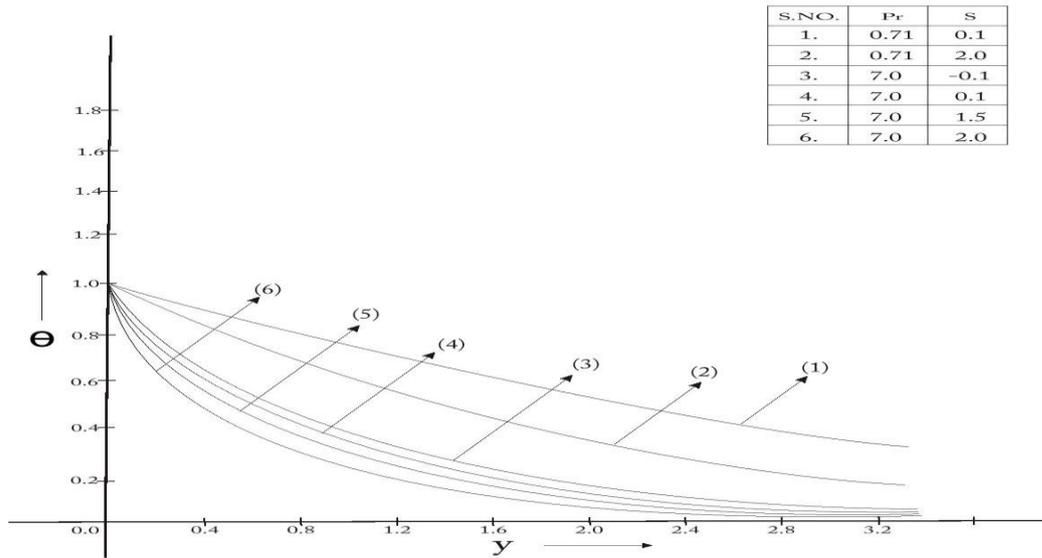


Figure 5. Temperature profile for different values of Pr and S.

Concentration profile is plotted in figure 6 and it is noticed that concentration is more for hydrogen ( $Sc = 0.22$ ) but less for propyl benzene ( $Sc = 2.62$ ). It is being observed that effect of Sorret number leads to rise in concentration profile.

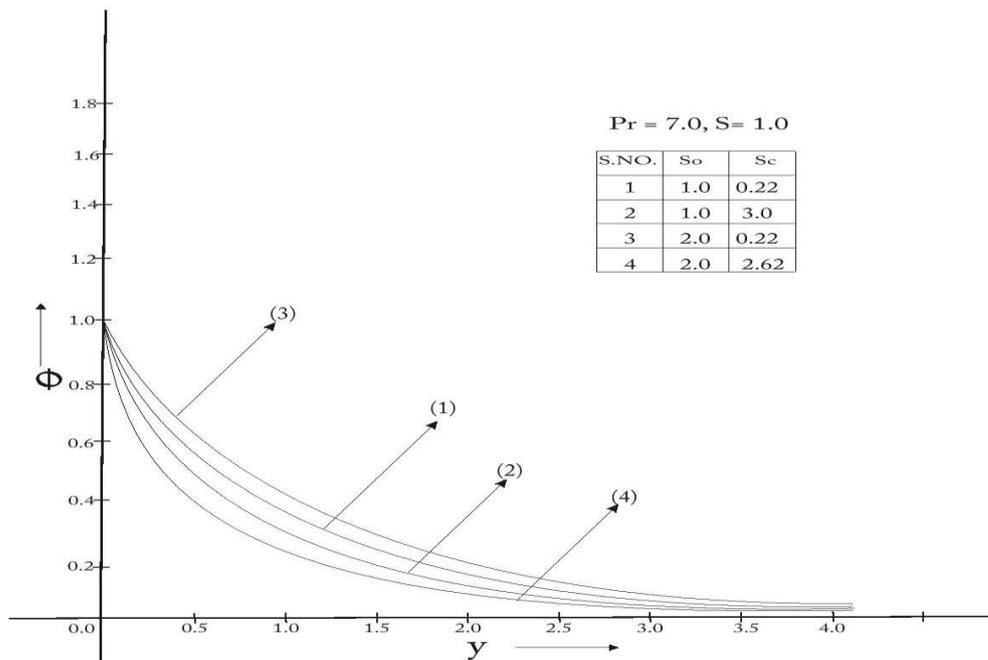


Figure 6. Concentration profile for different values of So and Sc.

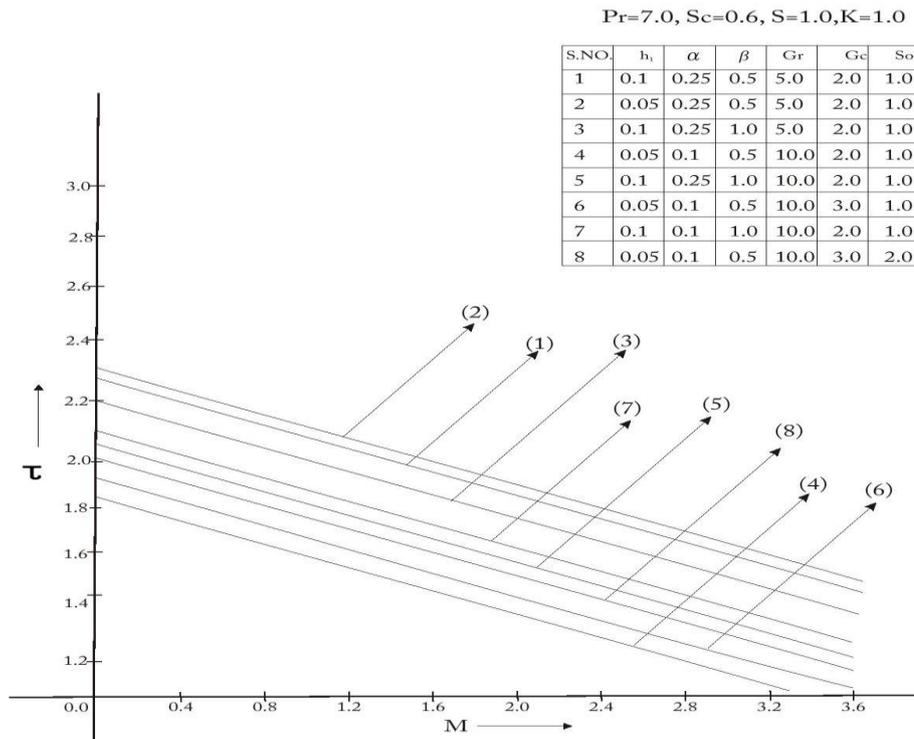


Figure 7. Skin friction for different values of  $h_1, \alpha, \beta, Gr, Gc$  and  $S_0$

In figure 7, dimensionless skin friction is plotted against  $M$  for different values of  $h_1, \alpha, \beta, Gr, Gc$  and  $S_0$ . It is being observed that increase in  $h_1, Gr, \alpha$  and  $\beta$  decreases the skin friction but increase in  $Gc$  and  $S_0$  increases the skin friction.

Important parameter namely Nusselt number are plotted in figure 8. it is being concluded that rate of heat transfer is more for Water ( $Pr = 7.0$ ) in Comparison with air ( $Pr = 0.71$ ). it is observed that increase in slip parameter increases the Nusselt number for both the basic fluids air and water.

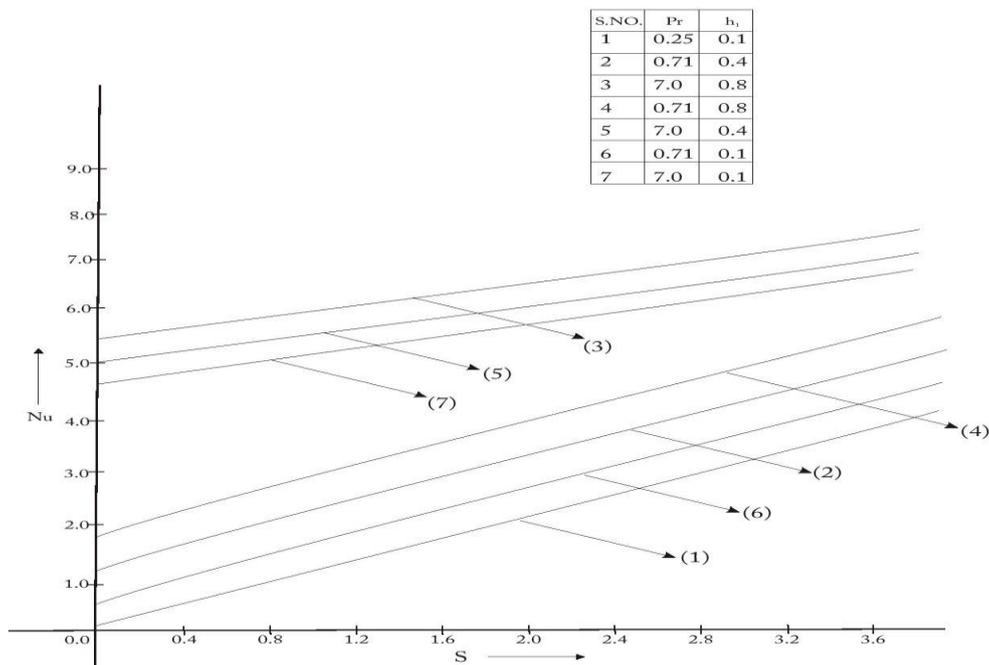


Figure 8. Nusselt number for different values of  $Pr$  and  $h_1$

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