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# Modal Analysis of a Multiple Cracked Cantilever Bar

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***ABSTRACT:*** *In this paper, we used the innovative technique recommended for the calculation of natural frequencies of a vibrating cantilever bar with a finite number of symmetric transverse open cracks. Additionally, the transfer matrix and the finite element method are considered to deal with the same problem. The procedure proposed is advanced by removal of numerical calculation of the high order element so that the computer time for calculating the natural frequencies is considerably reduced. Numerical analysis has been carried out to examine the effect of each single crack, the number of cracks and cantilever end condition bar for natural frequencies. The paper contains the cantilever bars with three cracks which permit the evaluation of natural frequencies predictable by these three methods.*

***Keywords:*** *vibration, multicracked bar, natural frequency, transfer matrix method,FEM.*

## I. INTRODUCTION

Cracks present a severe threat to the performance of structures and for this reason techniques allowing early finding and localization of cracks have been the subject of thorough investigations the last two decades. As a consequence, a variety of analytical, numerical and experimental studies now exist. A number of researchers have dealt with this topic by addressing either the direct or the inverse problem. In the modeling of structures with cracks, it is necessary to underscore the finite element method [1, 2]. This approach has no concurrence in application to large structures, but for specification of crack location in an element such as beam the analytical model of the element is more beneficial. In the analytical model of beams, crack is treated as a local change of stiffness (or flexibility) at a section of crack location. To model crack in this conception, Dimagoronas suggested the use of an equivalent rotational spring connecting both the sides of a beam at the crack position. Shekar [3] have studied the significance of dynamic behavior of rotors with double cracks and developed the finite element modeling of a rotor bearing system for the analysis of vibration characteristics of a rotor with two transverse open cracks. Tsai and Wang [4] have developed the theory for stepped cantilevers and for Timoshenko beam. Rizoset al. [5] have constructed the equation for cantilever beam by using the transverse model. Narkis [6] has given the equations for simply supported beam in both the cases of transverse and axial crack models. Ruotoloet al. [7] has studied the dynamic behaviour of a double-cracked beam and a rotor with two cracks. Shifrin and Ruotolo [8] was studied the beam with an arbitrary number n of cracks. who proposed a new method for evaluating natural frequencies of such a beam, that requires to calculate determinant of (n+2) order instead of (4n+4)-matrix determinant search as usually needed. Masoudet al. [9] have considered the case of axially loaded fixed cracked beam. Qinkai Han [10] have performed the dynamic analysis of a geared rotor-bearing structure with a breathing slant crack. A.K. Darpe et al. [11] have formulated the equation of motion of the rotor with a transverse surface crack with a bow and also attempted transient response and steady state analysis of the rotor. Nandwana and Maiti [12] have been established in a general form for all of classical boundary. As a result, the aim of this article is determine the longitudinal dynamic behavior of bars with several open cracks by extending the method previously proposed by Shifrin and Ruotolo [8] for the prediction of the transverse dynamic behaviour of multi-cracked beams. Some numerical examples complete the article, and comparisons are drawn with corresponding results provided by a transfer matrix approach and by the finite element method.

## II. THEORETICAL ANALYSIS

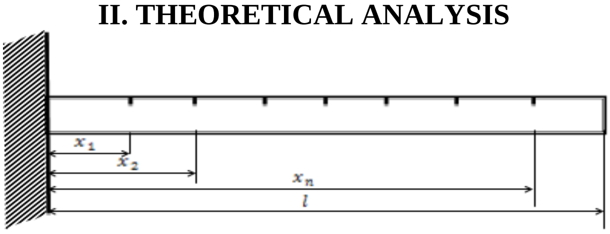


Fig.1. Multi-cracked cantilever bar

A bar with length $l$ and with n cracks is considered (Fig.1). It is assumed that cracks are located at points $x_1, x_2 .........x_n$ such that $0 < x_1 < x_2 < .......... < x_n < l$ Amplitudes of longitudinal displacement of the beam axis under time-harmonic vibration are denoted by $u_j(x)$ on the interval $x_{j-1} < x < x_j$

Where $j = 1, 2, ....., n+1,\ x_0 = 0$ and $x_{n+1} = l$ .

According to the approach proposed in Ref. [5], it is possible to division the total bar into $n+1$ bars joined by massless springs indicating the $n$ cracks. As a consequence, the equation of harmonic longitudinal oscillations of each bar, assumed with uniform cross-section is

$$EAu_j''(x) + \omega^2 \rho A u_j(x) = 0,\ j = 1, ......, n+1, x_{j-1} < x < x_{j,} \quad (1)$$

Where E is Young's modulus, $A$ is the area of the cross section, $\rho$ is the material density, and $\omega$ is a natural circular frequency. It is possible to introduce two conditions for each connection between two bars which, in correspondence with the location of the crack, impose continuity for the normal force and discontinuity for the longitudinal displacement of the bar in correspondence of the crack.

$$u_{j+1}(x_j) - u_j(x_j) = \Delta_j = EAcu_j'(x_j),\ j = 1, 2, ........, n, \quad (2)$$

Where $c_j$ is the flexibility of the $j_{th}$ translational spring which is function of the crack extent and bar width. In order to consider only the effect of the longitudinal vibrations, a double edge crack, symmetrical with respect to the longitudinal axis of the bar

$$C_j = \frac{2h(1-v^2)}{EA}\alpha_{tt}(s_j) \quad (3)$$

with $s_j = a_j / h$ where aj is the depth of the $j_{th}$ crack

The general solution can be written as,

$$u_0(x) = \alpha\cos(\lambda x) + \beta\sin(\lambda x) + \sum_{j=1}^{n} \Delta_j H(x - x_j)\left[\cos\lambda(x - xj) - 1\right] \quad (4)$$

Where $\alpha$ , $\beta$ are constants . By differentiating the previous function once it is possible to obtain the expression for $u_0''(x)$ at the cracks positions $x_i$

$$u_0'(x_i) = -\alpha\lambda\sin(\lambda x_i) + \beta\lambda\cos(\lambda x_i) + \sum_{j=1}^{n} \Delta_j N_{ij} \quad (5)$$

**1. Transfer matrix method**

In this section the technique of transfer matrix, for computing the natural frequencies of a multi-cracked bar in bending vibration, has been extended to define the natural frequencies of a bar with fixed- free end conditions.

The general solution for the $j_{th}$ segment of Eq. (1) is

$$u_j(x) = C_{1,j}\cos\lambda(x - x_{j-1}) + C_{2,j}\sin\lambda(x - x_{j-1}) \quad (6)$$

Consequently, variables at the right end of the bar can be expressed as a function of those at the left end:

$$\{Z(x_{n+1})\}_{n+1} = [T]_{n+1}[Q]_n[Q]_{n+1..............}[Q]_1\{Z(x_0)\}_1 = [Q]\{Z(x_0)\}_1 \quad (7)$$

$$\{B^0\}^T\{Z(x_0)\}_1 = 0,\ \{B^l\}^T\{Z(x_{n+1})\}_{n+1} = \{B^l\}^T[Q]\{Z(x_0)\}_1 = 0, \quad (8)$$

In order to determine the natural frequencies of the bar, the following equation must be solved:

$$\det\left[A(\omega, \{X\}, \{C\})\right] = B_1^0(B_1^lQ_{12} + B_1^lQ_{22}) - B_1^0(B_1^lQ_{11} + B_1^lQ_{21}) = 0 \quad (9)$$

For the following boundary conditions Eq. (9) becomes

$$fixed - free B_1^0 = B_1^l = 0 \det[\mathrm{A}] = \mathrm{B}_1^0\mathrm{B}_2^l Q_{22} = Q_{22} = 0 \quad (10)$$

Numerical results and discussion

In order to validate the procedures proposed in this paper,the dynamic behavior of a cantilever bar with three cracks has been simulated. The results are equated with those obtained using a finite element model with ten elements. The bar under analysis has the following mechanical properties: Length L=1m, Young's modulus $E$=2e+11N/m$^2$, Material density ρ = 7800kg/m$^3$, Poisson's ratio μ=0.33and the rectangular cross-section with Width b=0.03 m and Height h = 0.03m.In use of the method developed in the present paper for the same rod, numerical computation has been carried out and results were compared also with those given in reference [8]. The first three natural frequencies for cantilever bar, for the undamaged case, are listed in Table 1, allowing comparison of the results predicted by the continuous model with the finite element method.

Table.1. Natural frequencies for the undamaged cantilever bars calculated using the continuous model and FEM

| $f_n(H_z)$ | Continuous | FEM |
|---|---|---|
| 1 | 1631 | 1633 |
| 2 | 4884 | 4930 |
| 3 | 8137 | 8347 |

**3.1 Effect of crack position and depth**

Figure 2-4 provides the ratios of the first three natural frequencies for the bar with crack to the equivalent frequencies of the uncracked bar in fixed-free end condition of bar. The bar with a fixed end has a first crack at position x1 = 0.1 m and crack depth a1=0.003m, a second crack at position x2 =0.2 m and depth a2 =0.006m and third crack position x3=0.3 distance calculated from the fixed end and crack depth a3=0.009m. a relative crack depth a/h of 0.1, 0.2 and 0.3 have presented.

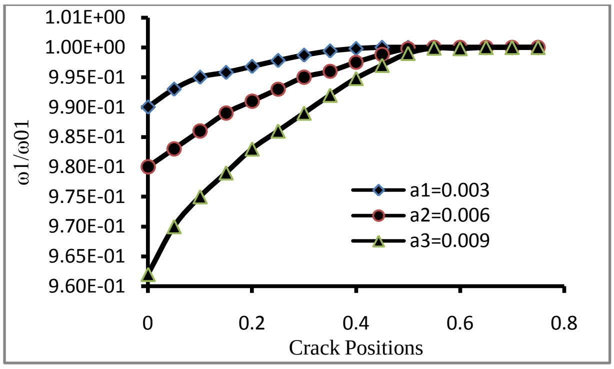


Fig. 2. Effect of the third crack on the first natural frequency for the cantilever bar

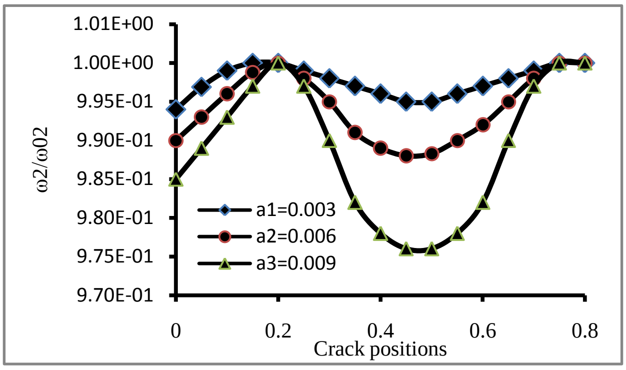


Fig.3. Effect of the third crack on the second natural frequency for the cantilever bar.

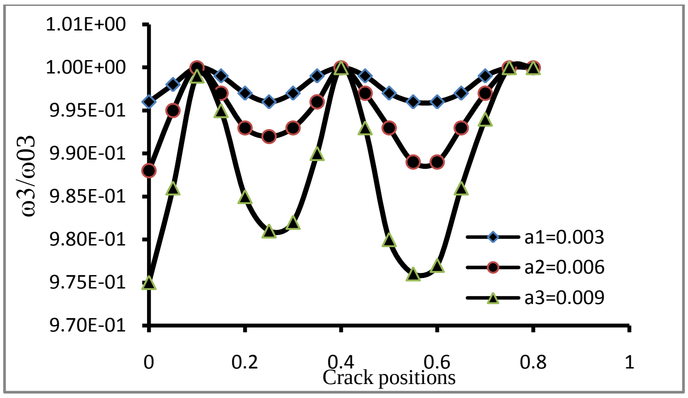


Fig. 4. Effect of the third crack on the third natural frequency for the cantilever bar

## III. CONCLUSIONS

In this paper natural frequencies of a cracked bar are evaluated by representing cracks as massless springs and considering a continuous mathematical model of the bar in longitudinal vibration. The frequency equation has been used to examine the effect of crack location and depth, number of cracks on the natural frequencies of a bar. One of the results obtained is that independent of the number of cracks there exists a set of positions in bar at which the presence of crack does not affect certain natural frequencies of the bar. These positions for a given frequency are called the critical points. Furthermore, the numerical computation shows also that increase in the number of cracks, in general, reduces all natural frequencies for cantilever bar. Finally a comparison of results obtained using the two methods based on a continuous model and the finite element method shows a very good agreement giving validity to the procedures proposed.

## REFERENCES

G. Gounaris and A. Dimagoronas, "A finite element of a cracked prismatic beams for structural analysis", *Computers and Structures* 28, pp.309-313, 1988.

B. S. Haisty and W. T. Springer, "A general beam element for use in damage assessment of complex structure", *Journal of vibration, Acoustics, Stress and Reliability in Design*, vol. 110, pp.389-394, 1988.

A.S. Sekhar, "Vibration characteristics of a cracked rotor with two open cracks", *Journal of Sound and vibration*, vol. 223, pp.497-512, 1999.

T. C. Tsai and Y. Z. Wang, "Vibration analysis and diagnosis of a cracked shaft",*Journal of Sound and vibration*, vol.192, pp.607-620, 1996

P. F. Rizos, N. Aspragathos and A. D. Dimaroganas,"Identification of crack location and Magnitude in a cantilever beam from the vibration modes", *Journal of Sound and vibration* ,vol. 138, pp.381- 388, 1990.

Y. Narkis, "Identification of crack location in vibrating simply supported beams",*Journal of Sound and vibration*,vol.172, pp.549-558, 1994.

R. Ruotolo, C. Surace and C. Mares, "Theoretical and experimental study of the dynamic behavior of a double-cracked beam", *Proceedings of 14th International Modal Analysis Conference*, pp. 1560-1564, 1996.

E. I. Shifrin and R. Ruotolo, "Natural frequencies of a beam with an arbitrary number of cracks",*Journal of Sound and vibration* vol.222, pp.409-423, 1999

S. Masoud, M. A. Jarran and M. Al-Maamory, "Effect of crack depth on the natural frequency of a prestressed fixed-fixed beam", *Journal of Sound and vibration* ,vol.214, pp. 201-212, 1998

Qinkai Han, Jingshan Zhao and Fulssei Chu, "Dynamic analysis of a geared rotor system considering a slant crack on the shaft", *Journal of Sound and Vibration*,vol.331 ,pp. 5803–5823, 2012.

A.K. Drape, K. Gupta and A. Chawla, "Dynamics of a bowed rotor with a transverse surface crack", *Journal of Sound and Vibration*, vol. 296, pp. 888–907, 2006.

B.P.Nandwana and S. K.Maiti,"Detection of the location and size of a crack in stepped cantilever beams based on measurements of natural frequencies", *Journal of sound and vibration*, vol.203,pp.435-446, 1997.