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Prediction of Comprehensive Strength and Water Absorption of Polystyrene Lightweight Concrete Using Osadebe's Regression Model

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Abstract
Polystyrene light weight concrete in Nigeria are faced with the problem of mix proportioning to meet the compressive strength and water absorption requirements. This paper developed new models for predicting the 7th, 14th, 21th, and28th day compressive strength and water absorption of polystyrene lightweight concrete using Osadebe's regression model which expresses the response as a function of the proportions of the mixture components. The models were formulated using existing data and were validated using the F-statistics and probability plots. A comparison of the models and the existing ones based on Osadebe's simplex lattice model. The models have a wider range of application and are more user friendly. Adequacy of the models was confirmed using F-statistics and normal probability plot. Four component were developed to determine the responses to a given mix and the mixes that gave a desired response value. The effect of the partial replacement of polystyrene with coarse aggregate was also studied. Cement and water has the greatest effect on the properties. The structural properties of the polystyrene lightweight concrete reduce when polystyrene was partially replaced with coarse aggregate. The optimum replacement waswith a decrease in compressive strength of concrete. It was concluded that the usage of expanded polystyrene lightweight concrete should be encouraged in Nigeria especially to reduce the weight of structure resting on the foundation.
Key words: Water, Absorption, Polystyrene, Lightweight, Concrete, Regression Model

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I. Introduction

Partial replacement of polystyrene with granite in the production of lightweight concrete is now a common practice in Nigeria. The practice is highly encouraged as it has been shown by many researchers to reduce the structural weight of the concrete. Concrete works in Nigeria are confronted with the problem of mix proportioning to meet the minimum requirements of Nigeria Industrial standard for concrete. There is therefore the need to develop models that could be used to predict the structural characteristics of polystyrene lightweight concrete (Babu and Babu, 2003). Such models, if developed using statistical approaches, can also be used for studying mixture components' interactions and optimization. Anya and Osadebe(2005) developed models for predicting the compressive strength and water absorption through the usage of a statistical approach ofScheffe's design. Through the models, they studied the mixture components' interactions. However, the components were expressed in pseudo ratios which have to be transformed to their actual real component ratios (Anyaogu and Ezeh, 2013). Expressing the components' proportions in pseudo ratios enabled the individual component contribution to be evaluated but made the model less user friendly. Also the models can be applied only to mixes within the simplex. Concrete are extensively used in construction of buildings and other structures for the service of man. Being a very important construction material, it is expedient that its quality and durability be of paramount concern to designers, builders, and other users of structures made with them. As the demand for natural sand continue to increase on daily basis, arising from the building and construction of new infrastructure and expansion of existing ones, the construction industries of most developing countries are compelled to identify alternative materials in order to lessen or eliminate the demand for common known concrete. Polystyrene are some of the materials that have been identified as suitable alternatives and are now being used either partially to replace coarse aggregate. Their consideration as alternatives to weight in concrete which has led to the collapse of many structures due to the weight of concrete on high structures.Mulla and Shelake (2016) lack of polystyrene could led to continuous increase in the cost of construction. Hence utilizing polystyrene either partially or wholly as substitutes for granite in the production of lightweight concrete can significantly bring down the cost of construction. The alarming rate of unrestricted sand mining which damages the ecosystem of natural habitants of organisms living on the riverbeds, affects fish breeding and migration, spells disaster for the conservation of many birds' species and increases saline water in the rivers. Recently, in most major cities across Nigeria, manufacturers of lightweight concrete have resorted to utilizing a combination of polystyrene to replace granite, and at other occasions, a combination of polystyrene, sand and granite for the production of lightweight concrete used in the construction of buildings and structures. Producers of lightweight concrete that utilize these materials use arbitrary mixes and the strength of concrete produced using of polystyrene, as replacement for granite can be guaranteed (Neville, 2011). The appropriate mix of polystyrene and granite or the three is known. Drawing from the forgoing therefore, the study addresses these gaps and concerns raised. The objective of this work is to develop friendly models with wider range of application for predicting the 28th day compressive strength and water absorption of polystyrene lightweight concrete by the use of Osadebe's regression model in which the components' proportions are expressed in actual proportions.

Osadebe's model.

To predict comprehensive strength and water absorption of polystyrene, Osadebe (2003) expressed the response,***y***as a function of the proportions of the constituents of the mixture, ***Zi.*** The sum of all the proportions like as in mixture experiments must add up to 1.

That is,

$$Z_1 + Z_2 + \cdots + Z_q = \sum_{i=1}^{q} Zi = 1 \quad (1)$$

Where q is the number of mixture components and $Z_i$the proportion of the component in the mixture.

Osadebe assumed that the response Y is continuous and differentiable with respect to its predictors and can be expanded in the neighbourhood of a chosen point Z_o using Taylor's series.

$$Z(0) = (Z_1^{(0)}, Z_2^{(0)}, \dots, Z_q^{(0)})^r \quad (2)$$

$$Y(z) = \sum_{m=0}^{q} F^m \, (Z)^{(0)} \big(Z_i - Z^{(0)}\big) \quad (3)$$

Expanding to second order

$$Y(Z) = F(Z^{(0)}) + \sum_{i=1}^{q} \frac{\partial f\big(Z^{(0)}\big)}{\partial Z_i} \big(Z_i - Z^{(0)}\big) + \frac{1}{2!} \sum_{i=1}^{q-1} \sum_{i=1}^{q} \frac{\partial^2 f(Z^0)}{\partial Z_i \partial Z_i} (Z_i - Z_i{}^{(0)})(Z_i - Z_i{}^{(0)})$$

$$+ \sum_{i-1}^{q} \frac{[\![\partial^2 f(Z]\!]^{(0)})}{\partial Z_i} \big(Z_i -^{(0)}\big) \quad (4)$$

For convenience, the point Z^0 can be taken as the origin without loss in generality of the formulation and thus;

$$Z_1{}^{(0)} = Z_1{}^{(0)} + Z_2{}^{(0)} + Z_3{}^{(0)} + \dots, Z_q{}^{(0)} = 0 \quad (5)$$

Let:

$$b_0 = F(0), \qquad b_i = \frac{\partial F(0)}{\partial Z_i}, \qquad b_{ij} = \frac{\partial^2 F(0)}{2i \partial Z_i \partial j}, \qquad b_{ii} = \frac{\partial^2 F(0)}{2i \partial Z_i^2} \quad (6)$$

Substituting Equation (2.3) into Equation (1) gives:

$$Y(Z) = b_0 + \sum_{i-1}^{q} b_i Z_i + \sum_{i \le j \le q}^{q} b_{ij} Z_i Z_j + \sum_{i-1}^{q} b_{ii} Z_i^2 \quad (7)$$

Multiplying Equation (1) by b_0 gives the expression:

$$b_0 = b_0 Z_1 + b_0 Z_2 + \cdots \dots + b_0 Z_q \quad (8)$$

Multiplying Equation (2.1) successively by $Z_1, Z_2 \dots Z_q$ and rearranging, gives respectively:

$$Z_1^2 = Z_1 - Z_1 Z_2 - \cdots \dots \dots + Z_1 Z_q$$

$$Z_2^2 = Z_2 - Z_1 Z_2 - \cdots \dots \dots - Z_2 Z_q$$

$$Z_q^2 = Z_1 - Z_1 Z_q - \cdots \dots \dots - Z_{(q-1)} \quad (9)$$

Substituting Equations (2.6) and (2.7) into Eq. (2.9) and simplifying yields

$$Y(Z) = \sum_{i-1}^{q} \beta_i Z_i + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i Z_j \quad (10)$$

$$\beta_i = b_0 + b_i \dots + b_{ii} \quad (11)$$

$$\beta_{ij} = b_{ij} - b_{ii} - b_{ij} \quad (12)$$

Equation (2.10) is Osadebe's regression model equation. It is defined if the unknown constant coefficients, β_i and β_{ij} are uniquely determined. If the number of constituents, *q*, is 4, and the degree of the polynomial, *m*, is 2 then the Osadebe regression equation is given as:

$$Y = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_4 Z_4 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{14} Z_1 Z_4 + \beta_{23} Z_2 Z_3 \quad (13)$$

$$+\beta_{24} Z_2 Z_4 + \beta_{34} Z_3 Z_4 \quad (14)$$

The number of coefficients, *N* is now the same as that for the Scheffe's (4, 2) model given by (15) as:

$$N = C_m^{(q+m-1)} = C_m^{(4+2-1)} = 10$$

$$\mathrm{N} = \frac{(q+m-1)!}{M!\,([\![q+m-1]\!]-M)!} = \frac{(q+m-1)!}{m!\,(q-1)!}$$

$$\frac{(4+2-1)!}{2!\,(4-1)!} = \frac{5!}{2!\,3!} = 10$$

Coefficients of Osadebe's Regression Equation

The least number of experimental runs or independent responses necessary to determine the coefficients of the Osadebe's regression coefficients is *N*. Let $y^{(k)}$be the response at point *k* and the vector corresponding to the set of component proportions (predictors) at point *k* be $Z^{(k)}$.

That is: $Z^{(k)} = \left(Z1^{(k)}, Z2^{(k)}, \ldots . Zq^{(k)}\right)$ (16)

Substituting the vector of Equation (13) into Equation (9) gives:

$$Y^{(k)} = \sum_{i=1}^{q} \beta_i \, {Z_i}^{(k)} + \sum_{i \le j \le q}^{q} \beta_{ij} \, {Z_i}^{(k)} {Z_j}^{(k)} \quad (17)$$

Where k=1, 2… N

Substituting the predictor vectors at each of the *N* observation points successively into Equation (9) gives a set of *N* linear algebraic equations which can be written in matrix form as:

Zβ=Y (18)

Where

$\beta$is a vector whose elements are the estimates of the regression coefficients.

The Coefficient of the Regression Equation

Let the K^{th} response (compressive strength for the serial number k) be $y^{(k)}$ and the vector of the corresponding set of variables be $Z^{(k)} = [Z_1^{(k)}, Z_2^{(k)}, Z_3^{(k)}, Z_4^{(k)}]^T$ (19)

Substitution of the above vector in eqn. 2.16 for k = 1,2, …, 10, generates the following system of ten linear algebraic equations in the unknown coefficients βi and βij.

$Y^{(k)} = \sum \beta i Z i^{(k)} + \sum \beta ij Z i^{(k)} \, Zj^{(k)}$

$1 \; 1 \le i \le 4$ and k = 1, 2, 3… 10

Let

$$[y^{(k)}] = \begin{bmatrix} y(1) \\ y(2) \\ \cdot \\ \cdot \\ \cdot \\ y(10) \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{10} \end{bmatrix} \begin{bmatrix} & Z_1^{(1)}, Z_1^{(2)}, \ldots, Z_1^{(10)}, & \\ Z_2^{(1)}, & & Z_2^{(2)}, \ldots, Z_2^{(10)}, \\ Z_3^{(1)} Z_4^{(1)}, Z_3^{(2)}, Z_4^{(3)} & \ldots, Z_3^{(10)} Z_4^{(10)} & \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{10} \end{bmatrix} = \begin{bmatrix} & Z_1^{(1)}, Z_1^{(2)}, \ldots, Z_1^{(10)}, & \\ Z_2^{(1)}, & & Z_2^{(2)}, \ldots, Z_2^{(10)}, \\ Z_3^{(1)} Z_4^{(1)}, Z_3^{(2)}, Z_4^{(3)} & \ldots, Z_3^{(10)} Z_4^{(10)} & \end{bmatrix} \begin{bmatrix} y(1) \\ y(2) \\ \cdot \\ \cdot \\ \cdot \\ y(10) \end{bmatrix} \quad (21)$$

Where, Z_i^k = Weight of variable i in k^{th} experimental run

Total weight of variable i in k^{th} experimental run

Model selection and validation (test for goodness)

The polynomial equations are fitted to the data obtained from the experimental results. It should be noted that though the experiment may have been designed based on a polynomial of degree *n*, the selected model may be of a lesser degree or have less number of terms than the original equation. This is true especially for the simplex lattice models. For example, Simon (2003) recommended a linear model for predicting the 28th day compressive strength of high strength concrete though the design had been based on a quadratic model.

Model selection and test of goodness for the augmented simplex lattice model

Selection of models and model validation are best done using an analysis of variance (ANOVA) table. The variances of interest are the Sum of Squares among blends or Sum of Squares due to Regression (*SSR*), Sum of Squares due to Residuals (*SSE*) and the Total Sum of Squares (*SST*). Others include Lack-of-fit sum of squares and pure error sum of squares (due to replicates).*SSR* shows the variation among the various blends while *SSE* is the variation among the replicate observations. *SST* is the sum of *SSR* and *SSE*. The formulae for computing the various sum of squares (Montgomery and Runger, 2003) are:

$$SSR = \sum_{i=1}^{M} (\hat{y}_i - \bar{y})^2 \quad (22)$$

$$SSE = \sum_{i=1}^{M} (\hat{y}_i - \bar{y})^2 \quad (23)$$

$$SST = \sum_{i=1}^{M} (\hat{y}_i - \bar{y})^2 \quad (24)$$

Matrix expression for the various sums of squares (Cornell, 2002):

$$SSR = p^1X^1y - \frac{(1^1y)^2}{M} \quad (25)$$

$$SSE = y^1y - p^1X^1y \quad (26)$$

$$SST = \beta^1X^1y - \frac{(1^1y)^2}{M} \quad (27)$$

Where:

M is the total number of observations at all the points.

$\hat{y}_i$is the predicted value of $y_i$using the fitted model.

$\hat{y}$ is the overall average of the observations, that is $(\sum y_i)/M$

β, y and X are matrices as previously described in this paper.

The associated degrees of freedom for *SSR, SSE*, and *SST* are *(p-1)*, (*M-p*) and *(M-1)* respectively. *p*is the number of terms in the model equation.

For model selection, an *F* test is performed on each of the models of interest. The highest order model with significant terms will be chosen (Simon, 2003, Cornell,2002).

Here the null and alternate hypotheses being tested are:

Ho: the tested term is not significant

Ha: the tested term is significant

The *F* statistic is given by the formula:

$$F = \frac{\textit{Mean square Regression (MSR)}}{\textit{Mean square Error (MSE)}} \quad (28)$$

$$F(\mathrm{p}-1), (M-p), \alpha. \quad (29)$$

The mean squares are obtained by dividing the respective sums of squares by their associated degrees of freedom. The *F* statistic is significant if the calculated value exceeds the value obtained from the *F* distribution table for the appropriate numerator and denominator degrees of freedom and confidence level, α

The logic behind the use of the *F* test in determining whether the regression relationship is statistically significant is based on the development of two independent estimates of the variance, σ2 (Anderson et al, 2007). *MSE*provides an independent estimate of σ2 and if the model is adequate, *MSR* provides another independent estimate of σ2.

Model validation

To validate the model or test for goodness of fit the hypotheses tested are:

Ho: lack-of-fit is significant

Ha: lack-of-fit is not significant

To carry out a lack-of-fit test, one needs to determine the external estimate of the population variance, σ2. Cornell (2002) described the procedure for testing the lack-of-fit when the estimate of σ2 is available and when it is not available. The procedure for the latter is summarized below. σ 2 is estimated using the mean square error (*MSE*) from the analysis of the fitted model This is made possible by collecting replicate observations at some of the design points. The replicate observations are then used to estimate σ2. Such an estimator of σ2 is called pure error. Additional observations are collected inside the simplex. These are called control or check

points. The observed data values at the check points are then compared to the values of the response that the fitted model predicts at the same check points. If the predictions made with fitted model are close to the values of the response observed at the check points, then the model is assumed adequate, otherwise the model is inadequate and said to fail owing to lack-of-fit. Again, the hypothesis is tested using the *F* statistic.
SSE is partitioned into Lack-of-fit sum of squares (SS_{Lof}) and pure error (due to replicates) sum of squares (SS_{pe}).
That is:

$$SSE = SS_{Lof} + SS_{pe} Or SS_{Lof} = SSE - SS_{pe} \quad (30)$$

II. Materials And Methods

The materials involved in this research were water, cement, sand and coarse aggregate.
Cement: The ordinary Portland cement (OPC) was used in which the composition and properties were in compliance with the Nigerian standard organization's defined standard of cement for concrete production. It was obtained from UNICEM.
Sand: The sand were collected from the local Qua-river in Akpabuyo Local Government Area of Cross River State, however, this research restricted washed sand and the sand was collected to ensure that there was no allowance for deleterious materials contained in the sand. As presented in Table 1, granite of 5-20mm maximum size was used. Proper inspection was carried out to ensure that it was free from deleterious materials.
Polystyrene: The expanded polystyrene (EPS) were obtained from a local distributor in Owerri. All the materials were collected and stored in a dry area in the laboratory, thereafter chemical analysis was conducted on the polystyrene to determine the elemental composition of EPS as showed in Table 2.

Modelling

The experimental design applied in this research was a Completely Random Design (CRD) with a mixture and process variables obtained from the laboratory on compressive strength and water absorption as showed in Table 1 and 2 using the set of mixture variables with three compounds of sand, cement, water and coarse aggregates. The process variableswere definedas size of the polystyrene concrete and proportion of polystyrene concrete which substitute coarse aggregates and the two variables were simultaneously adjusted.

Components Transformation of Polystyrene

The component proportion ratio was transformed to real the component ratios used for the blending of the polystyrene. The relationship between the real component ratios is indicated in equation 38:

R = AP (38)

From Equ. 38, R is a vector containing the real ratios of the components, P is a vector containing the component proportion ratios and *A* is a transformation matrix which can be obtained from trial mixes given as:

0.35 1 1 2
A = 0.44 1 1.5 3
0.45 1 2 3

The element of each column of [A] represents the components proportions at the vertex in the following order of water (Z_1), cement (Z_2), sand (Z_3) and Coarse aggregate (Z_4).
The totality of all the polystyrenes were blended using a crush machine. The aggregates were used in their dry condition and batching was by weight. Manual mixing was employed. Here, the entire polystyrene were put into backs and sook in a portable water inside a container, thereafter the polystyrenes were blend and cured in open air for 28 days by sprinkling them with water twice daily.

Table 1: Actual (Zi) and Pseudo (Xi)components for Osadebe's (4, 2) Simplex Lattice

Mix ratios					Component's fraction			
S/N	S_1	S_2	S_3	S_4	Z_1	Z_2	Z_3	Z_4
1	0.35	1	1	2	0.08	0.229885	0.229885	0.45977
2	0.44	1	1.5	3	0.074	0.16835	0.252525	0.505051
3	0.45	1	2	3	0.07	0.155039	0.310078	0.465116
4	0.5	1	3	6	0.048	0.095238	0.285714	0.571429
5	0.43	1	2	4	0.058	0.13459	0.269179	0.538358
6	0.48	1	2.5	5	0.053	0.111359	0.278396	0.556793
7	0.51	1	4	6	0.044	0.086881	0.347524	0.521286
8	0.33	1	3	5	0.035	0.107181	0.321543	0.535906

9	0.55	1	2	5	0.064	0.116959	0.233918	0.584795
10	0.6	1	2.5	6	0.059	0.09901	0.247525	0.594059
CONTROL								
11	0.55	1	1	2	0.121	0.21978	0.21978	0.43956
12	0.6	1	1.5	3	0.098	0.245902	0.245902	0.491803
13	0.44	1	2	4	0.059	0.134409	0.268817	0.537634
14	0.5	1	2.5	5	0.056	0.277778	0.277778	0.555556
15	0.4	1	3	6	0.038	0.096154	0.288462	0.576923
16	0.43	1	3.5	6.5	0.038	0.306212	0.306212	0.568679
17	0.35	1	4	7	0.028	0.080972	0.323887	0.566802
18	0.51	1	4.5	7.5	0.038	0.333087	0.333087	0.555144
19	0.48	1	4.8	7.6	0.035	0.072046	0.345821	0.54755
20	0.47	1	5	8	0.032	0.345543	0.345543	0.552868

Legend:

Z_1 = Actual water ratio

Z_2 = Actual cement quantity

Z_3 = Actual sand quantity

Z_4 = Actual coarse aggregate quantity

Table 2: Data obtained on compressive strength using Osadebe's Mix

Mix	Age of Cubes (Days)	Force (Kn)	Stress (N/mm^2)	Average Stress (N/mm^2)
1.00	3	323.15	14.36	14.30
		326.28	14.50	
		315.88	14.04	
	7.00	525.12	23.34	23.24
		530.21	23.56	
		513.31	22.81	
	14.00	662.46	29.44	29.32
		668.88	29.73	
		647.56	28.78	
	21.00	731.13	32.49	32.36
		738.22	32.81	
		714.69	31.76	
	28.00	799.80	35.55	35.39
		807.55	35.89	
		781.81	34.75	
2.00	3	291.09	12.94	
		277.12	12.32	13.03
		311.47	13.84	
	7.00	473.02	21.02	
		450.32	20.01	21.18
		506.14	22.50	
	14.00	596.73	26.52	
		568.09	25.25	26.72
		638.51	28.38	
	21.00	658.59	29.27	
		626.98	27.87	29.49
		704.70	31.32	
	28.00	720.44	32.02	
		685.86	30.48	32.25
		770.89	34.26	
3.00	3.00	329.47	14.64	14.27
		345.94	15.38	
		287.48	12.78	
	7.00	439.29	19.52	19.02
		461.25	20.50	
		383.31	17.04	
	14.00	554.18	24.63	23.99

		581.88	25.86	
		483.56	21.49	
	21.00	611.63	27.18	26.48
		642.20	28.54	
		533.69	23.72	
	28.00	669.07	29.74	28.97
		702.52	31.22	
		583.81	25.95	
4.00	3.00	329.47	14.64	14.27
		345.94	15.38	
		287.48	12.78	
	7.00	439.29	19.52	19.02
		461.25	20.50	
		383.31	17.04	
	14.00	554.18	24.63	23.99
		581.88	25.86	
		483.56	21.49	
	21.00	611.63	27.18	26.48
		642.20	28.54	
		533.69	23.72	
	28.00	669.07	29.74	28.97
		702.52	31.22	
		583.81	25.95	
5.00	3.00	371.67	16.52	12.65
		236.77	10.52	
		245.12	10.89	
	7.00	536.85	23.86	18.27
		342.00	15.20	
		354.06	15.74	
	14.00	677.26	30.10	23.04
		431.45	19.18	
		446.66	19.85	
	21.00	747.46	33.22	25.43
		476.17	21.16	
		492.96	21.91	
	28.00	817.66	36.34	27.82
		520.89	23.15	
		539.26	23.97	
6.00	3.00	371.67	16.52	12.65
		236.77	10.52	
		245.12	10.89	
	7.00	536.85	23.86	18.27
		342.00	15.20	
		354.06	15.74	
	14.00	677.26	30.10	23.04
		431.45	19.18	
		446.66	19.85	
	21.00	747.46	33.22	25.43
		476.17	21.16	
		492.96	21.91	
	28.00	817.66	36.34	27.82
		520.89	23.15	
		539.26	23.97	
7.00	3.00	148.37	6.59	6.95
		164.34	7.30	
		156.55	6.96	
	7.00	214.31	9.53	10.04
		237.38	10.55	
		226.13	10.05	
	14.00	270.36	12.02	12.67
		299.46	13.31	
		285.27	12.68	
	21.00	298.39	13.26	13.98
		330.50	14.69	
		314.84	13.99	
	28.00	326.41	14.51	15.29
		361.54	16.07	
		344.41	15.31	
8.00	3.00	174.46	7.75	7.62
		171.35	7.62	

		168.23	7.48	
	7.00	252.00	11.20	11.00
		247.50	11.00	
		243.00	10.80	
	14.00	317.91	14.13	13.88
		312.23	13.88	
		306.55	13.62	
	21.00	350.86	15.59	15.32
		344.60	15.32	
		338.33	15.04	
	28.00	383.82	17.06	16.75
		376.96	16.75	
		370.11	16.45	
9.00	3.00	264.81	11.77	11.34
		257.18	11.43	
		243.78	10.83	
	7.00	382.50	17.00	16.39
		371.48	16.51	
		352.13	15.65	
	14.00	482.54	21.45	20.67
		468.63	20.83	
		444.22	19.74	
	21.00	532.56	23.67	22.82
		517.21	22.99	
		490.27	21.79	
	28.00	582.58	25.89	24.96
		565.79	25.15	
		536.31	23.84	
10.00	3.00	160.44	7.13	7.40
		168.39	7.48	
		170.57	7.58	
	7.00	231.75	10.30	10.69
		243.23	10.81	
		246.38	10.95	
	14.00	292.36	12.99	13.48
		306.84	13.64	
		310.81	13.81	
	21.00	322.67	14.34	14.88
		338.64	15.05	
		343.03	15.25	
	28.00	352.97	15.69	16.28
		370.45	16.46	
		375.25	16.68	
CONTROL				
11.00	3.00	342.69	15.23	14.69
		420.58	18.69	
		228.20	10.14	
	7.00	495.00	22.00	21.22
		607.50	27.00	
		329.63	14.65	
	14.00	624.46	27.75	26.77
		766.38	34.06	
		415.83	18.48	
	21.00	689.19	30.63	29.54
		845.83	37.59	
		458.94	20.40	
	28.00	753.92	33.51	32.31
		925.27	41.12	
		502.04	22.31	
12.00	3.00	381.63	16.96	14.01
		342.69	15.23	
		221.35	9.84	
	7.00	551.25	24.50	20.24
		495.00	22.00	
		319.73	14.21	
	14.00	695.42	30.91	25.53
		624.46	27.75	
		403.35	17.93	
	21.00	767.51	34.11	28.18
		689.19	30.63	

		445.16	19.78	
	28.00	839.60	37.32	30.82
		753.92	33.51	
		486.97	21.64	
13.00	3.00	220.22	9.79	10.83
		268.17	11.92	
		242.31	10.77	
	7.00	318.10	14.14	15.64
		387.36	17.22	
		350.00	15.56	
	14.00	459.16	20.41	21.41
		501.63	22.29	
		484.13	21.52	
	21.00	529.68	23.54	24.29
		558.76	24.83	
		551.19	24.50	
	28.00	600.21	26.68	27.18
		615.89	27.37	
		618.25	27.48	
14.00	3.00	163.61	7.27	7.90
		178.35	7.93	
		191.04	8.49	
	7.00	236.32	10.50	11.41
		257.62	11.45	
		275.94	12.26	
	14.00	298.13	13.25	14.39
		325.00	14.44	
		348.11	15.47	
	21.00	329.03	14.62	15.88
		358.69	15.94	
		384.19	17.08	
	28.00	359.93	16.00	17.37
		392.38	17.44	
		420.28	18.68	
15.00	3.00	149.54	6.65	6.49
		142.37	6.33	
		146.01	6.49	
	7.00	243.00	10.80	10.54
		231.35	10.28	
		237.26	10.55	
	14.00	306.55	13.62	13.30
		291.85	12.97	
		299.32	13.30	
	21.00	338.33	15.04	14.68
		322.10	14.32	
		330.34	14.68	
	28.00	370.11	16.45	16.06
		352.36	15.66	
		361.37	16.06	
16.00	3.00	134.65	5.98	6.28
		144.00	6.40	
		145.22	6.45	
	7.00	218.81	9.73	10.20
		234.00	10.40	
		235.98	10.49	
	14.00	276.04	12.27	12.87
		295.20	13.12	
		297.70	13.23	
	21.00	304.65	13.54	14.21
		325.80	14.48	
		328.56	14.60	
	28.00	333.27	14.81	15.54
		356.40	15.84	
		359.42	15.97	
17.00	3.00	132.76	5.90	6.09
		139.54	6.20	
		138.46	6.15	
	7.00	215.73	9.59	9.89
		226.76	10.08	
		225.00	10.00	

	14.00	272.15	12.10	12.47
		286.06	12.71	
		283.85	12.62	
	21.00	300.36	13.35	13.77
		315.71	14.03	
		313.27	13.92	
	28.00	328.57	14.60	15.06
		345.37	15.35	
		342.69	15.23	
18.00	3.00	140.34	6.24	6.05
		131.86	5.86	
		136.18	6.05	
	7.00	228.05	10.14	9.83
		214.27	9.52	
		221.30	9.84	
	14.00	287.69	12.79	12.40
		270.31	12.01	
		279.18	12.41	
	21.00	317.52	14.11	13.69
		298.33	13.26	
		308.12	13.69	
	28.00	347.34	15.44	14.97
		326.35	14.50	
		337.06	14.98	
19.00	3.00	119.08	5.29	5.85
		138.88	6.17	
		137.10	6.09	
	7.00	193.50	8.60	9.51
		225.68	10.03	
		222.80	9.90	
	14.00	244.11	10.85	12.00
		284.70	12.65	
		281.06	12.49	
	21.00	269.41	11.97	13.24
		314.21	13.96	
		310.20	13.79	
	28.00	294.72	13.10	14.49
		343.72	15.28	
		339.33	15.08	
20.00	3.00	117.69	5.23	5.61
		145.38	6.46	
		115.37	5.13	
	7.00	191.25	8.50	9.11
		236.25	10.50	
		187.47	8.33	
	14.00	241.27	10.72	11.49
		298.04	13.25	
		236.50	10.51	
	21.00	266.28	11.83	12.68
		328.93	14.62	
		261.02	11.60	
	28.00	291.29	12.95	13.88
		359.83	15.99	
		285.53	12.69	

From the Table 1, the experimental design of the actual (Z_i) components of Osadebe'smodel was tested. As transformed from equation (5), the actual component was used to measure out the quantities water (Z_1), cement (Z_2), sand (Z_3) and coarse aggregate (Z_4) for polystyrene lightweight concrete in accordance with their respective ratios for compressive strength. Twenty-eight days was adopted based on twenty (20) mix. The two point loading system was used under which the specimen failed was recorded. The entire component was cast in concrete mould of size 150 x 150 x 150mm. Thereafter, the concrete cubes were cured in a curing tank for 28 days and were crushed using universal testing machines. The compressive strength was based on the following equation:

$$\text{Compressive strength} = \frac{\text{Compressive load of cube at failure (N)}}{\text{Cross sectional area of mould (mm}^2\text{)}} \qquad (31)$$

Also, additional equation for compressive strength and water absorption was obtained from the bitters extraction in the laboratory.

III. Result And Discussion

The responses of compressive strength of Osadebe component proportion model and response are presented in Table 3.

Experimental method for Compressive strength and Water Absorption

The second degree model (*Equation.* 33), was fitted to the data set of the 20 compressive strength and water absorption test responses at 95% confidence limit (α=0.05 level of significant) using [23]. The One-Way Analysis of Variance (ANOVA) was used to compare the lack-of-fitted compressive strength of polystyrene lightweight concrete as shown in table3. The normal probability plot of the regression standardized residual is shownin Figure 1; while the cox response trace plot is presented in Figure 2; and the variation of compressive strength in both linear and quadratic are adequately represented in Figure 3 and 4; while the variation of water absorption is shown in Figure 5 and 6 respectively.

Equation

Compressive strength:

In accordance to Table 3, the polynomial coefficients were determined and the model equation established and tested for goodness using the *F* statistic.

$$\beta_1 = -21405,\ \beta_2 = 5425.1,\ \beta_3 = 19656,\ \beta_4 = -8403.9,\ \beta_{12} = 21054,\ \beta_{13} = 25965,\ \beta_{14} = 1544200,\ \beta_{23} = -53865,\ \beta_{24} = -72119,\ \beta_{34} = 5775.9 \quad (32)$$

Thus the resulting regression equation for Osadebe's second degree model is:

$$\hat{y} = -21405Z_1 + 5425.1Z_2 + 19656Z_3 - 8403.9Z_4 + 21054Z_1Z_2 + 25965Z_1Z_3 + 1544200Z_1Z_4 - 53865Z_2Z_3 - 72119Z_2Z_4 + 5775.9\ Z_3Z_4 \quad (33)$$

Table 3 shows insignificant lack-of-fit, on the p-value for lack-of-fit being 0.00 which is less than 0.05. Furthermore *equation*33,of the analysis of variance was used to compare the lack-of-fit of compressive strength. The comparison yielded an F-ratio of 139.428 which is greater than .05 level of significant. This thereforeimplies that equation 33 is adequate for predicting the 28^{th} day of compressive strength using Osadebe's model.

The normal probability of regression standardized residual in Figure 1, show that the residuals fall inside the reference line, indicating that the data follow in a normal distribution. The cox response trace plot in Figure 2, show relative significant deviation in the proportion of fitted compressive strengthof Z_1 (0.23), Z_2 (0.27), Z_3 (0.31) Z_4 (0.19) which is higher than 0.05 respectively. This simply implies that, the prediction of compressive strength at 28 days is adequate for expanded polystyrene lightweight concrete using Osadebe's model.

Water absorption:

For water absorption, Table 4 of the polynomial coefficients were determined and the model equation and bitter's values were established and tested for goodness using the *F* statistic. From the *equation* 35, analysis of variance was used to compare the lack-of-fit of water absorption, the comparison yielded an F-ratio of 2.908 that is higher than 0.05 level of significant. The model also shows that there is insignificant lack-of-fit, the p-value for lack-of-fit being 0.56^b which is higher than 0.05 and is adequate for predicting the 28^{th} day water absorption of polystyrene lightweight concretes.

$$\beta_1 = 175.12,\ \beta_2 = -3.3217,\ \beta_3 = -197.67,\ \beta_4 = 145.3,\ \beta_{12} = -139.48,\ \beta_{13} = -407.4,\ \beta_{14} = -2079,\ \beta_{23} = 587.59,\ \beta_{24} = 921.07,\ \beta_{34} = -270.69 \quad (34)$$

Thus the resulting regression equation for Osadebe's second degree model is:

$$\hat{y} = 175.12Z_1 - 3.3217Z_2 - 197.67Z_3 + 145.3Z_4 - 139.48Z_1Z_2 - 407.4Z_1Z_3 - 2079Z_1Z_4 + 587.597Z_2Z_3 + 921.07Z_2Z_4 - 270.69Z_3Z_4 \quad (35)$$

Figure 3, show the normal probability plot of water absorption which the residuals fall inside the reference line, indicating that the data follow in a normal distribution. The cox response trace plot in Figure 4, show relative significant deviation in the proportion of fitted compressive strength of Z_1 (0.41), Z_2 (0.23), Z_3 (0.11) Z_4 (0.25) which is higher than 0.05 respectively. This simply implies that, at 28 days of prediction, water absorption is adequate for expanded polystyrene lightweight concrete using Osadebe's model.

Table 3: One-Way Analysis of Variance (ANOVA) of compressive strength test results of expanded polystyrene lightweight concrete

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	Z_1	-21405	486.652	-.658	-.503	.625
	Z_2	5425.1	157.098	6.789	3.384	.006
	Z_3	19656	63.370	2.641	1.756	.107
	Z_4	-8403.9	851.852	-.018	-.041	.968
	$Z_1 * Z_2$	21054	1507.346	-.930	-.871	.403
	$Z_1 * Z_3$	25965	1118.944	1.223	.796	.443
	$Z_1 * Z_4$	1540000	344.379	.200	.232	.821
	$Z_2 * Z_3$	-53865	243.446	-.149	-.135	.895
	$Z_2 * Z_4$	-72119	263.024	-8.213	-4.522	.001
	$Z_3 * Z_4$	5775.9	486.652	-.658	-.503	.625
Model	**Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**	
1	Regression	10180.817	9	1131.202	139.428	.000[c]
	Residual	89.245	11	8.113		
	Total	10270.062[d]	20			

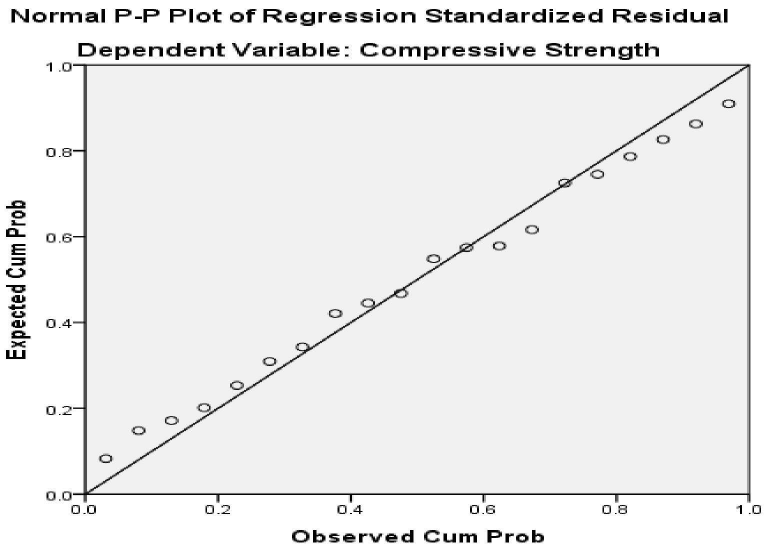

Figure 1: Compressive Strength Normal Probability Plot of Regression Standardize Residual

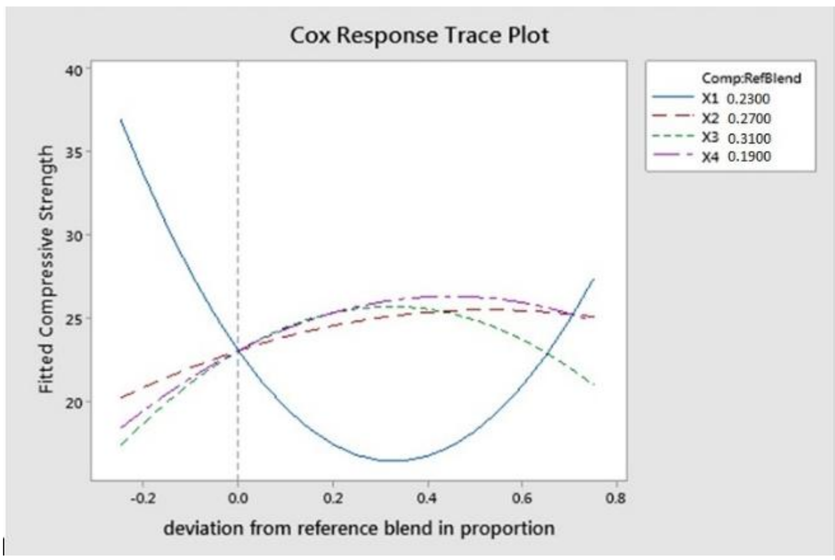

Figure 2: Compressive Strength Cox Response Trace Plot

Table 4: One-Way Analysis of Variance (ANOVA) of water absorption test results of expanded polystyrene lightweight concrete

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta		
1	Z1	175.12	17.241	1.836	.815	.434
	Z2	-3.3217	145.942	2.029	.248	.809
	Z3	-197.67	83.295	-1.759	-.176	.864
	Z4	145.3	66.700	.596	.243	.813
	Z1 * Z2	-139.48	131.226	-2.909	-1.445	.179
	Z1 * Z3	-407.4	147.233	-2.266	-.871	.404
	Z1 * Z4	-2079	170.285	-.030	-.010	.992
	Z2 * Z3	587.59	116.683	3.917	.895	.392
	Z2 * Z4	921.07	77.879	-2.802	-1.074	.308
	Z3 * Z4	-270.69	152.487	-4.077	-.548	.596
	Model	**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
	Regression	.569	9	.063	2.908	.056[b]
	Residual	.218	10	.022		
	Total	.787	19			
	Value	**Minimum**	**Maximum**	**Mean**	**Std. Deviation**	**N**
	Predicted Value	.7169	1.5525	.9095	.17311	20
	Residual	-.16163	.26456	.00000	.10700	20
	Std. Predicted Value	-1.112	3.714	.000	1.000	20
	Std. Residual	-1.096	1.794	.000	.725	20

Table 4 shows that the polynomial coefficients were determined and the model equation established and tested for goodness using the F statistic at 0.05 level. The conclusion, therefore, is that equation 35 is adequate for predicting the 28th day strength of expanded polystyrene lightweight concrete.

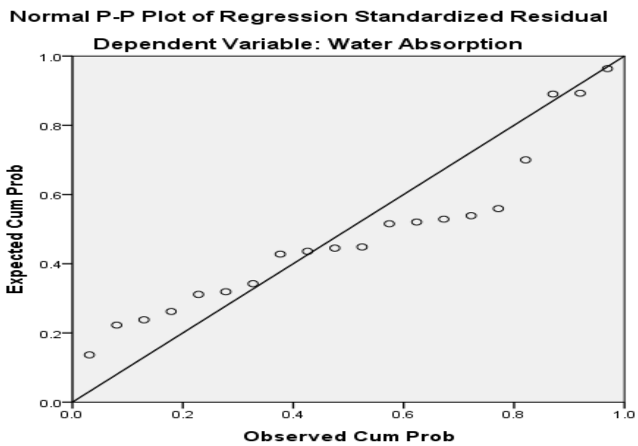

Figure 3: Water Absorption Normal Probability Plot of Regression Standardize Residual

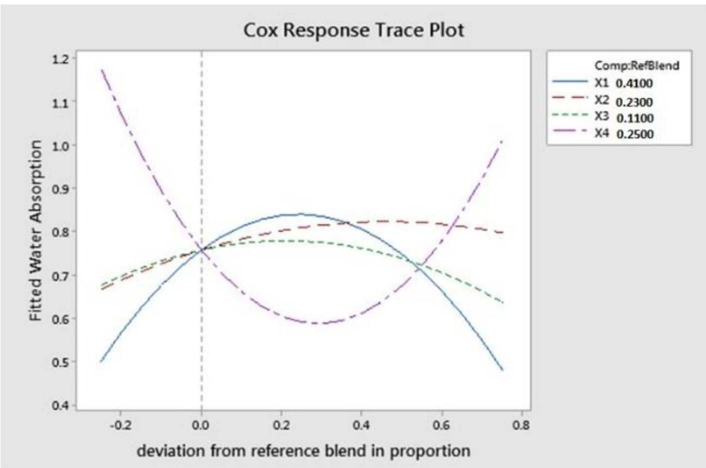

Figure 4: Water Absorption Cox Response Trace Plot

IV. Conclusion

The major objective of this research work was to determine the prediction of comprehensive strength and water absorption of polystyrene lightweight concrete using Osadebe's model. In using Osadebe's second degree regression equation, mix design model for a four component expanded polystyrene concrete was developed. This model predict the compressive strength of Expanded Polystyrene Lightweight Concrete (EPS) cube when the mix ratios are known and vice versa. All the prediction from the model were tested for lack-of-fit and were found adequate for predicting the various responses within the bounds of the simplex at 95% accuracy level using One-Way Analysis of Variance (ANOVA). Osadebe's model can be used to make predictions outside of the simplex, thus giving it a wider range of application.

1. The mathematical models developed are adequate to be used for optimization of polystyrene lightweight concrete (Polyc).
2. The strength values obtained in some of the optimised mixes are adequate for use as structural or load bearing concrete.
3. Recommendation is made for the practical application of these research work.

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