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Solution of Electromagnetic Propagation Problems in Shielded Transmission Lines Using the Eigenmode Projection Technique

Mohamed O. Ashour[1], Islam A. Eshrah[1]
[1](*Electronics and ElectricalCommunication Department, Faculty of Engineering, Cairo University, Egypt*)

Abstract:
An eigenmode projection technique is utilized to solve the problems ofthe electromagnetic wave propagation in shielded transmission lines. The technique isadopted to solve theproblems ofinfinite length rectangular shaped loaded lineswhere a fictitious canonical cavity surrounded by perfect electric surfaceis chosen to enclose the line and the fields inside are expanded in terms ofthe cavitysolenoidal and irrotational eigenmodes where they are considered as a complete set torepresent any vector field inside the cavity. The fields in Maxwell's equations inside theenclosed region are then expanded using the cavity eigenmodes. Finally, a set of equationsfor the eigenmodes are resulted by using thefields expansions in Maxwell's equations ofthe cavity where mode projections are done. This set of equations aresolved together toget the line dispersion curve and the propagating modes.
Key Word*: Eigenmodes; Resonance; Transmission Lines; Microstrip Lines; Coplanar Waveguides.*

I. Introduction

One of the early milestones in microwave engineering was the development of waveguide andother transmission lines for the low-loss transmission of power at high frequencies.Early RF and microwave systems relied on waveguides, two-wire lines, and coaxial linesfor transmission where the properties of these lineswere studied extensively[1,2] and exact mathematical derivations for the different propagating modes of fields, cut off frequencies, propagation constants, attenuation constants, and characteristic impedancewere introduced[3,4].Planar transmission lines in the form of stripline, microstrip lines, coplanar waveguides,and several other types of related geometries were then invented[5]. They have manyadvantages where they are lightweight, compact, cost effective, and capable of being easilyintegrated with active circuit devices to form microwave integrated circuits. However, dueto the geometries for these lines, their behaviors and analysis are very complicated[6].Shielded transmission linesin the form of shielded microstrip lines andshielded coplanar waveguides can be considered good models for their open equivalent lines provided that the dimensionsof the shield (closing walls) are adjusted to be equal to or greater than about 10 to 20 times the center conductorwidth[4].

For the analysis of the shielded structures, themodal expansion concept was widely used[7,8]. Also, it was integrated with someconventional numerical methods such as the finite-difference time-domain method (FDTD)[9], the finite element method (FEM)[10], and the integral equations using moment method(MoM)[11] aiming to produce new hybrid methods. Recently, an eigenmode projection technique (EPT) was introduced and used tosolve severalelectromagnetic problems: resonance[12],waveguide discontinuities[13], scattering[14], transient analysis of waveguide probe excitation[15] and electrostatic[16]problems. The focusof this paper is on the solution of the problems of shielded transmission lines using EPT.In Section II, the formulation of the eigenmode expansion method is presented. Section III covers theeigenmode solution of the infinite length rectangularshaped loaded transmission lines.

II. The Eigenmode Expansion Method

Throughout this section, the eigenmode expansion method is presented starting withexpanding the electric and magnetic fields as a series of different eigenmodes then expanding the derivatives of these fields by following arigorous mathematical framework to reach the Maxwell's equations for these fields as a series of eigenmodes.Finally,the complex differential equations of electric and magnetic fields are shown as a system oflinear equations, where the unknowns are the coefficients of the eigenmodes.

Eigenmode Expansion

According to Slater[17]and later the modification made by Kurokawa[18], the eigenmodeexpansion provides a representation for the electric and magnetic fields in anarbitrary-shaped cavity of volume $V_t$enclosed by a surface $S_t$which is assumed to bepartly perfect electric conducting $S_E$and partly perfect magnetic conducting $S_M$as shownin Figure 1, in terms of the cavity solenoidal and irrotational eigenmodes as

$$\mathbf{E}(\mathbf{r}) = \sum_n a_n \boldsymbol{E_n}(\boldsymbol{r}) + \sum_\alpha f_\alpha \boldsymbol{F_\alpha}(\boldsymbol{r}), \tag{1}$$

$$\mathbf{H}(\mathbf{r}) = \sum_n b_n \boldsymbol{H_n}(\boldsymbol{r}) + \sum_\lambda g_\lambda \boldsymbol{G_\lambda}(\boldsymbol{r}). \tag{2}$$

Where a_n, b_n, f_α and g_λ represent the coefficients of the cavity fields. $\mathbf{E}_n(\mathbf{r})$ and $\mathbf{H}_n(\mathbf{r})$ are the solenoidal electric and magnetic eigenmodes, respectively. $\mathbf{F}_\alpha(\mathbf{r})$ and $\mathbf{G}_\lambda(\mathbf{r})$ are the irrotational electric and magnetic eigenmodes, respectively. The solenoidal eigenmodes are coupled through the curl equations

$$\nabla \times \boldsymbol{E}_n(\boldsymbol{r}) = k_n \boldsymbol{H}_n(r), \qquad \nabla \times \boldsymbol{H}_n(\boldsymbol{r}) = k_n \boldsymbol{E}_n(\boldsymbol{r}). \tag{3}$$

and satisfy the homogeneous Helmholtz equation.

$$(\nabla^2 + k_n^2)\, \boldsymbol{E}_n(\boldsymbol{r}) = 0, \qquad (\nabla^2 + k_n^2)\, \boldsymbol{H}_n(\boldsymbol{r}) = 0. \tag{4}$$

The irrotational eigenmodes are represented by the scalar potentials (Φ_α, Ψ_λ)gradient through

$$l_\alpha\, \boldsymbol{F_\alpha}(\boldsymbol{r}) = \nabla\Phi_\alpha, \qquad w_\lambda\, \boldsymbol{G_\lambda}(\boldsymbol{r}) = \nabla\Psi_\lambda. \tag{5}$$

and those scalar potentials satisfy Helmholtzequation.

$$(\nabla^2 + l_\alpha^2)\, \Phi_\alpha = 0, \qquad (\nabla^2 + w_\lambda^2)\, \Psi_\lambda = 0. \tag{6}$$

where k_n, l_α,$w_\lambda$are the wavenumbers for thesolenoidal and irrotational electric and magnetic fields, respectively.

The cavity eigenmodes discussed above form a complete orthonormalset. Also, the projectionsbetween solenoidal and irrotational modes vanish[19].

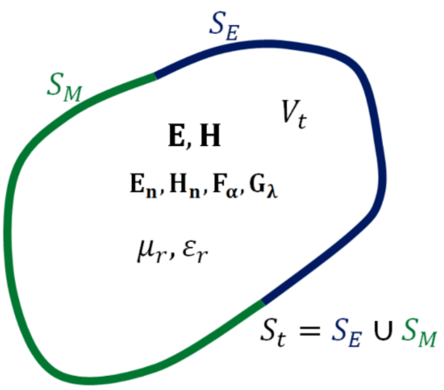


Figure1: An arbitrary-shaped cavity of volume $V_t$enclosed by a surface S_t, and its orthogonaleigen functions.

Field Derivatives Expansion

The expansions cannot be employed to determine the curl and the divergence termsdirectly. The curl operator over the electric and magnetic fields can be represented as[18,19,20]

$$\nabla \times \boldsymbol{E} = \sum_n (k_n a_n + \langle \boldsymbol{E}, \boldsymbol{H}_n \rangle_{St}) \boldsymbol{H}_n + \sum_\lambda (\langle \boldsymbol{E}, \boldsymbol{G}_\lambda \rangle_{St}) \boldsymbol{G}_\lambda \tag{7}$$

$$\nabla \times \boldsymbol{H} = \sum_n (k_n b_n + \langle \boldsymbol{H}, \boldsymbol{E}_n \rangle_{St}) \boldsymbol{E}_n + \sum_\alpha (\langle \boldsymbol{H}, \boldsymbol{F}_\alpha \rangle_{St}) \boldsymbol{F}_\alpha \tag{8}$$

Where$\langle \boldsymbol{X}, \boldsymbol{Y} \rangle_{St} = \oint_{St} \boldsymbol{X} \times \boldsymbol{Y}^{*}.\hat{\boldsymbol{n}}\, ds.$

Maxwell's Equations Expansion

By Substituting the expansions (2) and (7) into the Maxwell'sequation$\nabla \times \boldsymbol{E} = -j\omega\mu\ \boldsymbol{H}$and performing eigenmodeprojection with $\mathbf{H}_n$and $\mathbf{G}_\lambda$and substituting the expansions (1) and (8) into the Maxwell'sequation$\nabla \times \boldsymbol{H} = j\omega\epsilon\boldsymbol{E} + \boldsymbol{J}$and performing eigenmodeprojection with $\mathbf{E}_n$and $\mathbf{F}_\alpha$ using the fact that the modesare orthonormal, yields[18,20]

$$k_n a_n + \langle \boldsymbol{E}, \boldsymbol{H}_n \rangle_{St} = -j\omega\mu_o \left(\sum_{n'} b_{n'} \langle \mu_r \boldsymbol{H}_{n'}, \boldsymbol{H}_n \rangle + \sum_{\lambda'} g_{\lambda'} \langle \mu_r \boldsymbol{G}_{\lambda'}, \boldsymbol{H}_n \rangle \right) \tag{9}$$

$$\langle \boldsymbol{E}, \boldsymbol{G}_\lambda \rangle_{St} = -j\omega\mu_o \left(\sum_{n'} b_{n'} \langle \mu_r \boldsymbol{H}_{n'}, \boldsymbol{G}_\lambda \rangle + \sum_{\lambda'} g_{\lambda'} \langle \mu_r \boldsymbol{G}_{\lambda'}, \boldsymbol{G}_\lambda \rangle \right) \tag{10}$$

$$k_n b_n + \langle \boldsymbol{H}, \boldsymbol{E}_n \rangle_{St} = j\omega\epsilon_o \left(\sum_{n'} a_{n'} \langle \epsilon_r \boldsymbol{E}_{n'}, \boldsymbol{E}_n \rangle + \sum_{\alpha'} f_{\alpha'} \langle \epsilon_r \boldsymbol{F}_{\alpha'}, \boldsymbol{E}_n \rangle \right) + \langle \boldsymbol{J}, \boldsymbol{E}_n \rangle \quad (11)$$

$$\langle \boldsymbol{H}, \boldsymbol{F}_\alpha \rangle_{St} = j\omega\epsilon_o \left(\sum_{n'} a_{n'} \langle \epsilon_r \boldsymbol{E}_{n'}, \boldsymbol{F}_\alpha \rangle + \sum_{\alpha'} f_{\alpha'} \langle \epsilon_r \boldsymbol{F}_{\alpha'}, \boldsymbol{F}_\alpha \rangle \right) + \langle \boldsymbol{J}, \boldsymbol{F}_\alpha \rangle \quad (12)$$

Where$\langle \boldsymbol{X}, \boldsymbol{Y} \rangle = \int_{Vt} \boldsymbol{X}.\boldsymbol{Y}^* \, dv$. this term $\langle \boldsymbol{X}, \boldsymbol{Y} \rangle$denote the volumetric projection of thetwo vector functions Xand Y**.**By putting equations (9)-(12) in matrix form, we obtain

$$\boldsymbol{Ka} + \boldsymbol{Q}^{EH} = -j\omega\mu_o \, (\boldsymbol{M}^{HH}\boldsymbol{b} + \boldsymbol{M}^{GH}\boldsymbol{g}) \quad (13)$$

$$\boldsymbol{Q}^{EG} = -j\omega\mu_o \, (\boldsymbol{M}^{HG}\boldsymbol{b} + \boldsymbol{M}^{GG}\boldsymbol{g}) \quad (14)$$

$$\boldsymbol{Kb} + \boldsymbol{Q}^{HE} = j\omega\epsilon_o (\boldsymbol{W}^{EE}\boldsymbol{a} + \boldsymbol{W}^{FE}\boldsymbol{f}) + \boldsymbol{V}^{JE} \quad (15)$$

$$\boldsymbol{Q}^{HF} = j\omega\epsilon_o (\boldsymbol{W}^{EF}\boldsymbol{a} + \boldsymbol{W}^{FF}\boldsymbol{f}) + \boldsymbol{V}^{JF} \quad (16)$$

where $\boldsymbol{K}$is a diagonal matrix with diagonal elements k_n, $\boldsymbol{M}_{nn'}^{XY} = \langle \mu_r \boldsymbol{X}_{n'}, \boldsymbol{Y}_n \rangle$, $\boldsymbol{W}_{nn'}^{XY} = \langle \epsilon_r \boldsymbol{X}_{n'}, \boldsymbol{Y}_n \rangle$,$\boldsymbol{Q}_n^{XY} = \langle \boldsymbol{X}, \boldsymbol{Y}_n \rangle_{St}$, and $\boldsymbol{V}_n^{XY} = \langle \boldsymbol{X}, \boldsymbol{Y}_n \rangle$. **X** and **Y** can be **E**, **H**, **F**, **G**, or **J**.The above system of equations (13)-(16) represents the EPT core, where these equations are employed to solve the eigenmode problems for rectangularshaped electromagnetic problems in the next two sections.

III. Propagation Solution for Shielded Transmission Lines

It's required to evaluate the dispersion relation (the resonance frequencies) and themodes of a closed(shielded) rectangular transmission line loaded with dielectric, andlossy metal. The solution of the source-free wave propagation in a closed-boundary loadedstructure is similar to the cavity resonance problem[12,13,14,15,16]. Generally, EPT is used to study the resonance of arbitrary-shaped conducting cavitywith arbitrary dielectric loading as shownin Figure2a. The solution flow is as follows: asdepicted in Figure2b, a canonical cavity surrounded by either perfect electric (PE) or perfect magnetic (PM) orcomposite PE/PM surface is chosen, the canonical cavity solenoidaland irrotational eigenmodes are then derived analytically where they are considered as acomplete set to represent any vector field inside the cavity. The main problem is enclosedwithin this cavity as shown in Figure2c. The fields inside the enclosed region are expandedin terms of the derived cavity eigenmodes. The eigenvalue problem is finally formed byusing these expansions in Maxwell's equations of the cavity where mode projections aredone. It's worth mentioning that all PEC materials found in the problem are replacedwith highly conductive material, although practical conductors' loss tangent is frequencydependent, it is assumed that the used conductor has a constant loss tangent with highvalue over the frequency range of interest, which would be almost identicalwith thetheoretical PEC material at microwave frequencies.It is expected that the eigenvalues(resonance frequencies) in this case will be complex, and thespurious modes will be distinguished upon comparing the realand imaginary parts of the eigenvalue of the resulted modes.

Starting from the system of equations (13)-(16), All the surface integrals (integrals thatrepresent the coupling between the outer modes and the canonical cavity eigenmodes)vanish due to the fact of having zeroexternal fields (this is a closed problem and there isno outer modes).Also, dealing with a source-free medium withconstant permeability and making useof the case that the cavity eigenmodes form a complete orthonormalset.The system of equations(13)-(16) can be reduced to

$$\boldsymbol{Ka} = -j\omega\mu_o \boldsymbol{b} \quad (17)$$

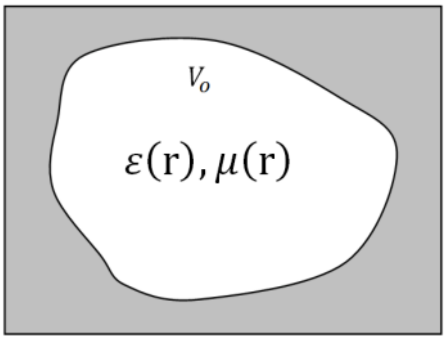
$$0 = -j\omega\mu_o \boldsymbol{g} \quad (18)$$

$$\boldsymbol{Kb} = j\omega\epsilon_o (\boldsymbol{W}^{EE}\boldsymbol{a} + \boldsymbol{W}^{FE}\boldsymbol{f}) \quad (19)$$

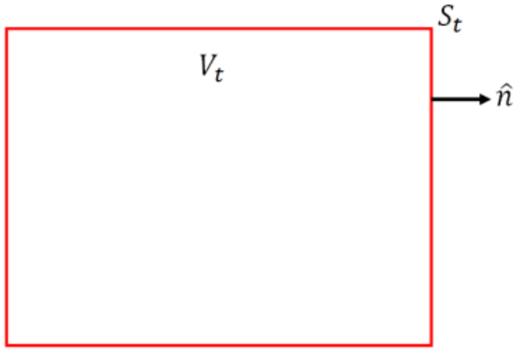
$$0 = j\omega\epsilon_o (\boldsymbol{W}^{EF}\boldsymbol{a} + \boldsymbol{W}^{FF}\boldsymbol{f}) \quad (20)$$

Solving equations (17)-(20) for the coefficients of the solenoidal electric field eigenmodes.

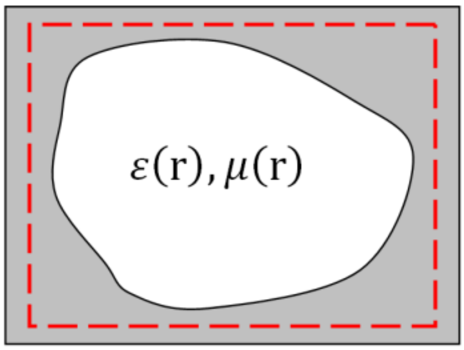
$$\boldsymbol{b} = -\frac{\boldsymbol{Ka}}{j\omega\mu_o} \quad (21)$$



(a) arbitrary-shaped conducting cavity with arbitrarydielectric loading.



(b) canonical cavity with known eigenmodes.



(c) enclosing the actual cavity with fictitiouscanonical cavity.

Figure2: Model development for the resonance problem using the eigenmode projectiontechnique.

$$\boldsymbol{f} = -(\boldsymbol{W}^{\boldsymbol{FF}})^{-1}\boldsymbol{W}^{\boldsymbol{EF}}\boldsymbol{a} \tag{22}$$

$$(\boldsymbol{W}^{\boldsymbol{EE}} - \boldsymbol{W}^{\boldsymbol{FE}}(\boldsymbol{W}^{\boldsymbol{FF}})^{-1}\boldsymbol{W}^{\boldsymbol{EF}})^{-1}\boldsymbol{K}^{2}\boldsymbol{a} - \omega^2\mu_o\epsilon_o\boldsymbol{a} = 0 \tag{23}$$

Equation (23) can be represented as

$$(\boldsymbol{\Omega} - k^2\boldsymbol{I})\boldsymbol{a} = 0 \tag{24}$$

Where $k^2 = \omega^2\mu_o\epsilon_o$and **I** is the identity matrix. So, the dispersion relation can beobtained by computing the matrix eigenvalues where they are used in determining thecorresponding wavenumbers. The modal field distribution can be determined by obtainingthe eigenvectors **a**. The other solenoidal and irrotational modes can be determinedusingequations (21),(22).

Results

First of all, the previous approach is verified for 2 special cases, the canonical caseof a PEC rectangular cavity partiallyfilled with dielectric, and the canonical case of a PEC rectangular cavity partiallyfilled withlossy metal. The results of the presented casesare based on the numbering scheme and the modes numbers required for convergence[21] wherefor any 1D problem, the maximum index used forvariations in v direction ($N_v^{\max}$) can be represented as

$$N_v^{\max} = \frac{1}{20\sigma_\epsilon^2}\frac{p}{\Delta_v} \tag{25}$$

Where p is the dimension of an object along the v direction, $\Delta_v$is the smallest dimensionof an object in the unit cell along the v direction, σ_ϵ^2 is a truncation parameter (to enhanceaccuracy decrease σ_ϵ^2), and the used number of modes is $N_v^{\max}$.In general, the numbering scheme of modes used on the following cases is determinedby considering all the combinations of different indices representing the mode variationsin x and y directions and convergence is achieved by increasing the maximum index usedin each direction.

The first case to be considered is a rectangular waveguide partially filled withdielectric material asshown in Figure 3. The waveguide has cross-sectional dimensions a= 22.86 mm, b = 10.16 mm, and the dielectric has d = 5.82 mm and ε_r= 10.The dispersion curve obtained usingthe EPT with the modes numbering scheme shown in Table 1 in comparison with theexact solution for the propagating modes is illustrated in Figure 4. In addition, Figure 5provides the electric field distribution using the EPT in comparison with the exact one forx = a/4 along the y-direction. It's to be noted that both figures show excellent agreementwith the analytical results for this case[4].

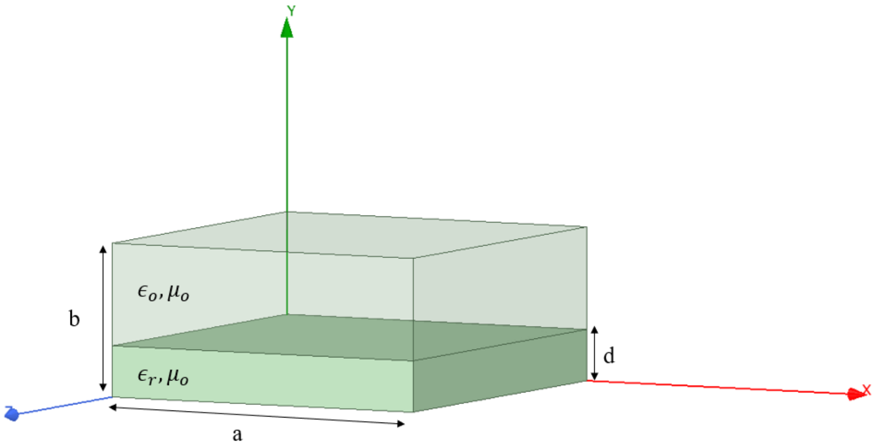


Figure3: Rectangular waveguide partially filled with dielectric material.

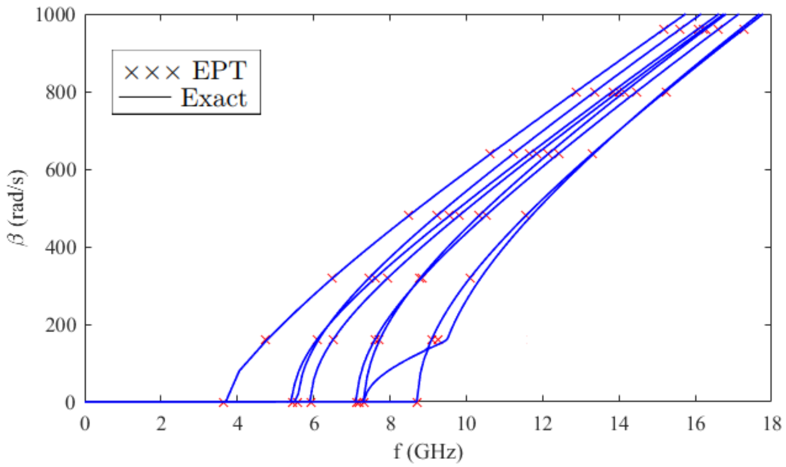


Figure4: Dispersion curve of a partially filled waveguide with a = 22.86 mm, b = 10.16 mm, d = 5.82 mm and ε_r= 10.

The second case to be considered is a rectangular waveguide partially filled with PEC asshown in Figure 6. The waveguide has cross-sectional dimensions a = 22.86 mm, b = 10.16mm, and the PEC has thickness d = 3.38mm. For this simple case, the results obtainedshould be the same as that of an empty waveguide with cross-sectional dimensions a×(b-d).By using the EPT with the modes numbering scheme shown in Table 2, The eigenvalues are complex where they can be separated into two sets: the first set contains themodes resonating in the

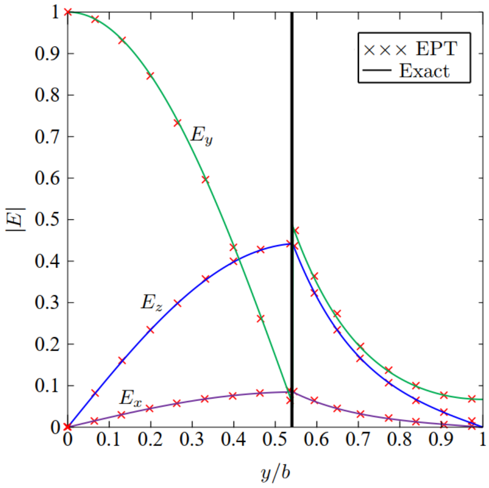


Figure5: Electric field distribution for the dominant mode along the y-direction at x = a/4 for a partially filled waveguide with a = 22.86 mm, b = 10.16 mm, d = 5.82 mm and ε_r= 10.

Table 1: Numbering scheme of the modes used to generate results in Figure 4 and Figure 5.

	n_x	n_y
Solenoidal TE	0,1	j
Solenoidal TM	1	j
Irrotational	1	j
Limits	$0 \leq j \leq 25$	

conductor and the other one contains the modes resonating insidethe cavity. The separation of the modes is done bycomparing the real and the imaginaryparts of the complex cut off wave number. Figure 7 shows the relation between the realpart and the imaginary part of the normalized cut off frequency for different loss tangent. The actual modes are the modes with high real part compared to the imaginary part fortanδ with large value (tanδ > 10^3)[16].The dispersion curve obtained using the EPT in comparison with the exact solutionfor the propagating modes is illustrated in Figure 8. In addition, Figure 9 provides theelectric field distribution using the EPT in comparison with the exact one. Both figuresshow excellent agreement with the exact results.

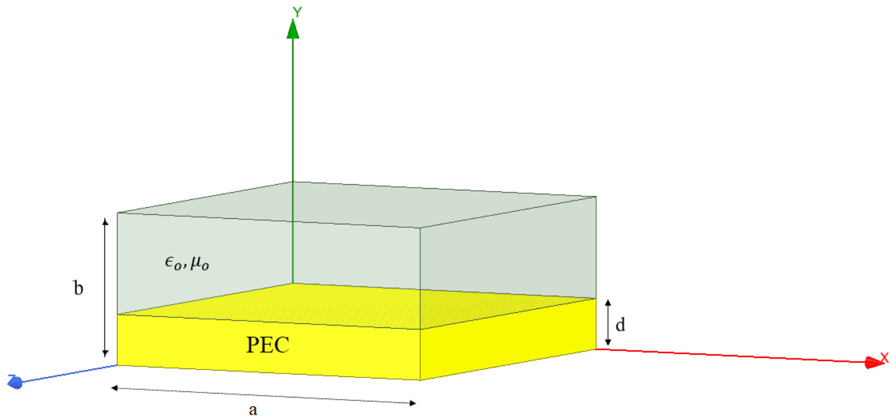


Figure6: Rectangular waveguide partially filled with PEC.

Second, shielded microstrip transmission line is a member of the family of the planar microwave transmission lines where it is similar to the basic microsrip line except for theenclosure and the side walls. The metallic enclosure will cover the entire structure asshown in Figure 10. The case to be considered is of a line which has cross-sectionaldimensions A = 12.7 mm and B = 12.7 mm, The patch has cross-sectional dimensions a =1.27 mm and t = 0.127 mm, The two side walls both have cross-sectional dimensions 2a =2.54 mm and a+t = 1.397 mm and the substrate has cross-sectional dimensions 6a = 7.62mm, a = 1.27 mm and εr = 2.56.Figure 11 compares the

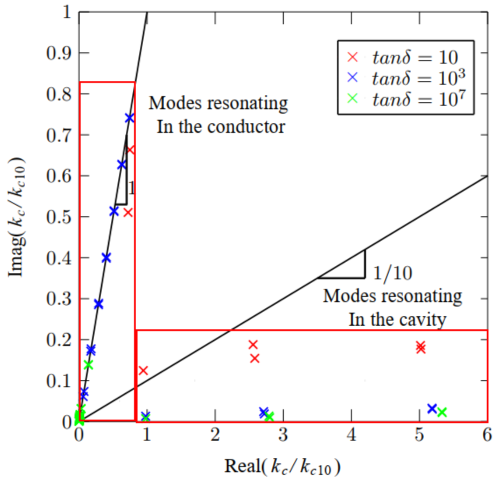


Figure7: Normalized complex cut off wave number for rectangular waveguide partially filled with PEC.

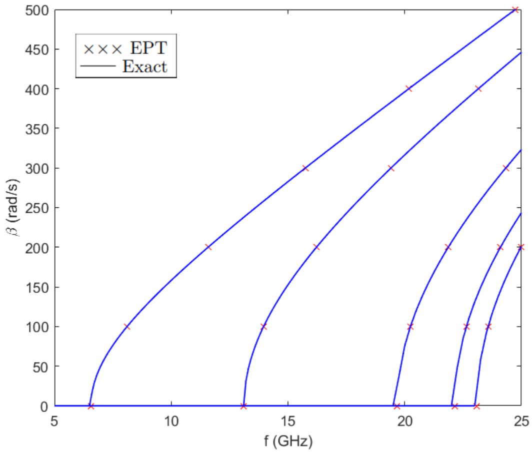


Figure8: Dispersion curve of a partially filledwaveguide with a = 22.86 mm, b = 10.16 mm and d = 3.38 mm.

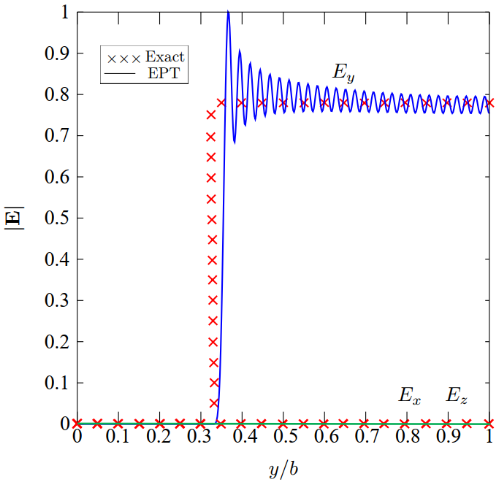


Figure9: Electric field distribution for the dominant mode for a partially filled waveguide with a= 22.86 mm, b = 10.16 mm, and d = 3.38 mm.

Table 2: Numbering scheme of the modes used to generate results in Figures 7, 8 and 9.

	n_x	n_y
Solenoidal TE	0,1	j
Solenoidal TM	1	j
Irrotational	1	j
Limits	$0 \le j \le 20$	

dispersion relation obtained using EPT with the numberingscheme of modes as shown in Table 3 and those obtained using the commercial solverHFSS. It is clear that excellent agreement is achieved for the different propagating modes.

Third, shielded coplananr waveguide problem is illustrated in Figure 12. The case to be considered is of a line which has cross-sectional dimensions A = 12.7 mmand B = 12.7 mm, The patch has cross-sectional dimensions a = 1.27 mm and t = 0.127mm, The two side walls both have cross-sectional dimensions 4a = 5.08 mm and a+t =1.397 mm and the substrate has cross-sectional dimensions 2a = 2.54 mm, a = 1.27 mmand ε_r= 2.56.Figure 13 compares the dispersion relation obtained using EPT with the numberingscheme of modes as shown in Table 4 andthose obtained using the commercial solverHFSS. It is clear that excellent agreement is achieved for the different propagating modes.

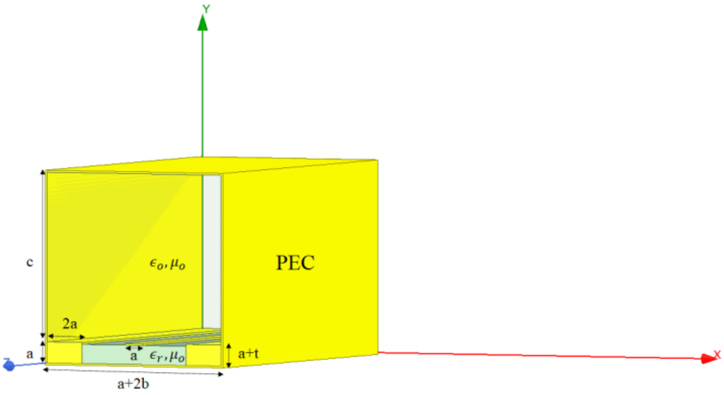


Figure10: Shielded microstrip transmission line.

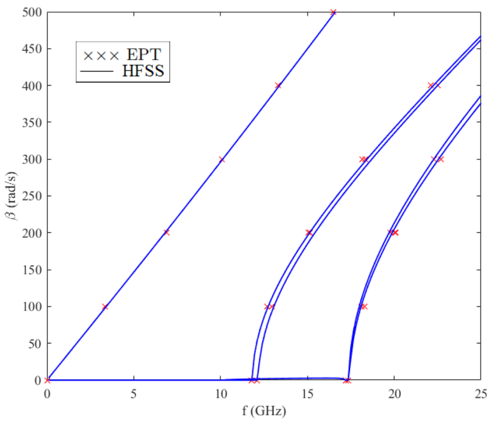


Figure11: Dispersion curve of shielded microstrip transmission line.

Table 3: Numbering scheme of the modes used to generate results in Figure 11.

	n_x	n_y
Solenoidal TE	i	j
Solenoidal TM	i	j
Irrotational	i	j
Limits	$0 \le i \le 25$	$0 \le i \le 35$

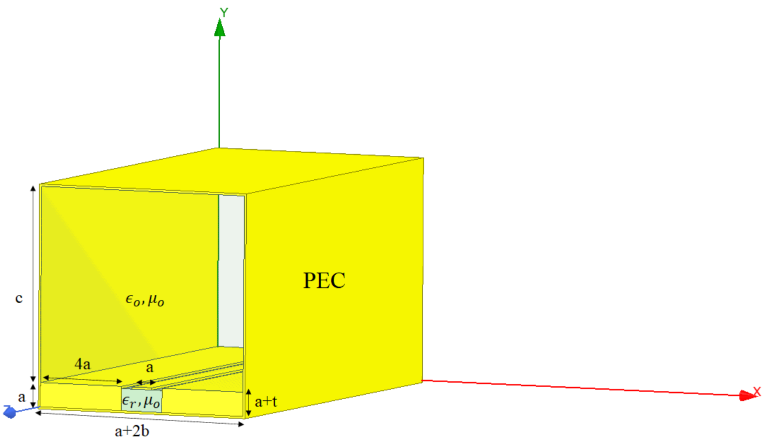


Figure12: Shielded coplananr waveguide.

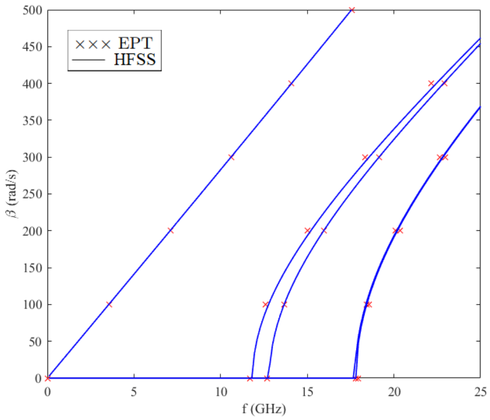


Figure13: Dispersion curve of shielded microstrip transmission line.

Table 4: Numbering scheme of the modes used to generate results in Figure 13.

	n_x	n_y
Solenoidal TE	i	j
Solenoidal TM	i	j
Irrotational	i	j
Limits	$0 \le i \le 30$ $0 \le i \le 35$	

IV. Conclusion

Throughout this work, an eigenmode projection technique was utilized to solve theproblems of the electromagnetic wave propagation in shielded transmission lines. The technique is adopted to solve the problems of infinite length rectangular shaped loaded lines.The EPT was shown to have many advantages compared to other conventional numerical techniques where it provides an automatic selection for the basis functions using complete orthogonalfunctions.It doesn't require an explicit subwavelength segmentation for the structure prior to the solution flow.The involved integrals have no singularity issues, and their kernels are typically sinusoidal for rectangular canonical structures.The integrals are also frequency independent, which means that all integrals will becalculated only one time through the solution flow to obtain the dispersion curve.Integrals are linear with permittivity, which allows for scaling of the integrals in caseof the presence of the variable dielectric constants.

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