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Two Approaches To Compressive Sensing With Differentiated Acquisition And Reconstruction

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Abstract:
This document presents two distinct approaches to compressive sensing for the acquisition and reconstruction of non-sparse signals. The first approach directly compresses the original signal using a measurement matrix, while the second approach first transforms the signal into a sparse representation before compression. The reconstruction phases employ the Orthogonal Matching Pursuit (OMP) algorithm to solve the minimization problem. The quality of reconstruction is evaluated using the Mean Squared Error (MSE), with both visual and numerical comparisons for different measurement sizes (M = 10 % to 90 %).
Keywords: *Compressive sensing, sparse signal, Orthogonal Matching Pursuit, Mean Squared Error*

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I. Introduction

Compressive sensing is a modern signal processing method that enables the reconstruction of a signal from a limited number of measurements. By leveraging this approach, it becomes possible to capture a signal in a compressed form right at the acquisition stage, significantly reducing the amount of data to be stored or transmitted while maintaining a satisfactory reconstruction quality.

II. Sparsity

Sparse signal
Definition

A finite-dimensional signal with discrete values, represented by a column vector x of size $N \times 1$, is said to be sparse when it contains only a limited number of significant or non-zero elements [1]. To aid understanding, Figure 01 and Figure 02 below present an example of a sparse signal and a non-sparse signal in the time domain.

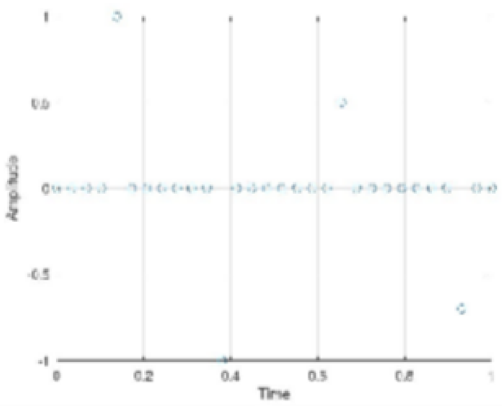

Figure 01. Sparse signal in the time domain

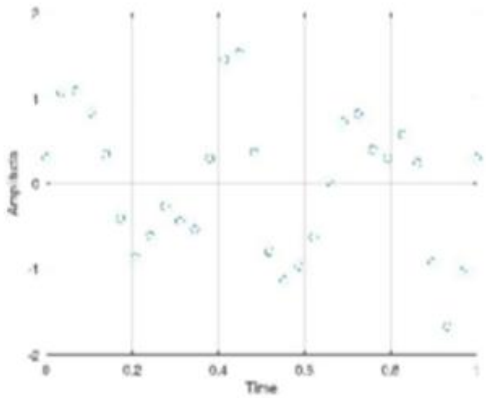

Figure 02. Non-sparse signal in the time domain

2.1.2. Measurement

The sparsity of a finite-dimensional signal with discrete values, represented by a column vector x of size $N \times 1$, can be measured using the l_0-norm, which corresponds to the number of non-zero elements in the vector [2]. From a mathematical perspective, this norm is defined as follows :

$$\|x\|_0 = \sum_{i=1}^{N} \delta(x_i) \quad (1)$$

Where :

- $\|x\|_0$ represents the l_0-norm of the column vector x
- $\delta(x_i)$ is an indicator function defined as follows :

$$\delta(x_i) = \begin{cases} 1 & \text{si} \quad x_i \neq 0 \\ 0 & \text{si} \quad x_i = 0 \end{cases} \quad (2)$$

- x_i represents the element of the column vector x located at the $i-thi$ row
- i is a natural number defined by :

$$i \in \{1, 2, \dots, N\}$$

- N is a natural number indicating the number of rows in the column vector x

2.2. Sparse representation

The sparse representation of a non-sparse, finite-dimensional, discrete-valued signal represented by a column vector x of size $N \times 1$ is a sparse, finite-dimensional, discrete-valued signal represented by a column vector α of size $N \times 1$, obtained by applying the following formula [3] :

$$\alpha = \psi x \quad (3)$$

Where :

ψ is a transformation matrix of size $N \times N$

III. Operation of compressive sensing

3.1. First approach

3.1.1. Acquisition or measurement phase

During this phase, the original non-sparse, finite-dimensional, discrete-valued signal represented by a column vector x of size $N \times 1$ is recorded in a compressed form in a measurement vector represented by a column vector y of size $M \times 1$, using the following formula :

$$y = \phi x \quad (4)$$

Where :

ϕ is a rectangular matrix of size $M \times N$, known as the measurement matrix $(M < N)$

3.1.2. Reconstruction phase

If the original non-sparse, finite-dimensional, discrete-valued signal represented by a column vector x of size $N \times 1$ has a sparse representation denoted by a column vector α of size $N \times 1$, then Equation (4) becomes :

$$y = A\alpha \tag{5}$$

Where :

- y is a column vector of size $M \times 1$, corresponding to the measurement vector
- A is a rectangular matrix of size $M \times N$, defined by :

$$A = \phi\psi^{-1} \tag{6}$$

During this phase, the reconstruction of the original signal is performed in two steps:

- First, the goal is to find the vector α that satisfies Equation (5). Since A is a rectangular matrix of size $M \times N$ with $M < N$, there exists an infinite number of vectors α that satisfy the equation [4]. However, assuming that α is sparse, the problem becomes finding the solution to the following minimization :

$$\hat{\alpha} = arg\ min \|\alpha\|_0 \text{ subject to } y = A\alpha \tag{7}$$

- Second, once $\hat{\alpha}$ is obtained, the original signal is reconstructed using the following formula :

$$\hat{x} = \psi^{-1}\hat{\alpha} \tag{8}$$

3.2. Second approach

3.2.1. Acquisition or measurement phase

During this phase, the compression of the original non-sparse, finite-dimensional, discrete-valued signal represented by a column vector x of size $N \times 1$ is not performed directly using Equation (4). Instead, a sparse representation of this signal is first obtained using Equation (3) [5], and then it is recorded in compressed form as a measurement vector represented by a column vector $M \times 1$ of size $M \times 1$, using Equation (5).

3.2.2. Reconstruction phase

The reconstruction phase of the second approach is identical to that of the first approach.

IV. Metric for evaluating reconstruction quality

In the context of signal reconstruction in compressive sensing, it is essential to assess the fidelity of the reconstructed signal $\hat{x}$ with respect to the original signal x. The Mean Squared Error (MSE) measures the squared distance between the components of the original signal x and those of the reconstructed signal $\hat{x}$ [6]. A low MSE value indicates that $\hat{x}$ is close to x, while a high value reflects an inaccurate reconstruction. From a mathematical perspective, it is defined as follows :

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \hat{x}_i\right)^2 \tag{9}$$

Where :

- N is the total number of components in the signal
- x_i and $\hat{x}_i$ represent the $i - th$ component of the original and reconstructed signals, respectively

V. Comparison between two compressive sensing approaches for the reconstruction of a non-sparse signal

In this section, the following points are specified :

- The discrete signal x of size 256×1 is obtained by sampling 256 points from the continuous signal $s(t) = cos(2\pi t) + sin(3\pi t)$.
- The measurement matrix ϕ of size $M \times 256$ is constructed by randomly selecting M rows from the identity matrix of size 256×256.

- M is expressed as a percentage (0 % corresponds to 0 rows, while 100 % corresponds to 256 rows).
- ψ is a Discrete Cosine Transform (DCT) matrix of size 256×256, where each element is computed using the following formula [7] :

$$\psi_{kn} = \begin{cases} \sqrt{\frac{1}{256}} cos\left[\frac{\pi}{256}\left(n + \frac{1}{2}\right)k\right] & si \quad k = 0 \\ \sqrt{\frac{2}{256}} cos\left[\frac{\pi}{256}\left(n + \frac{1}{2}\right)k\right] & si \quad k \geq 1 \end{cases} \tag{10}$$

Where :

- ψ_{kn} represents the element of the matrix ψ located at the $k - th$ row and $n - th$ column
- k and n are natural numbers defined by :

$$k \in \{0, 1, ..., 255\}$$

$$n \in \{0, 1, ..., 255\}$$

- The resolution of Equation (7) is carried out using the Orthogonal Matching Pursuit (OMP) algorithm [8]
- The Mean Squared Error (MSE) is computed using Equation (*9)*

For $M = 10$ %, Figure 03 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

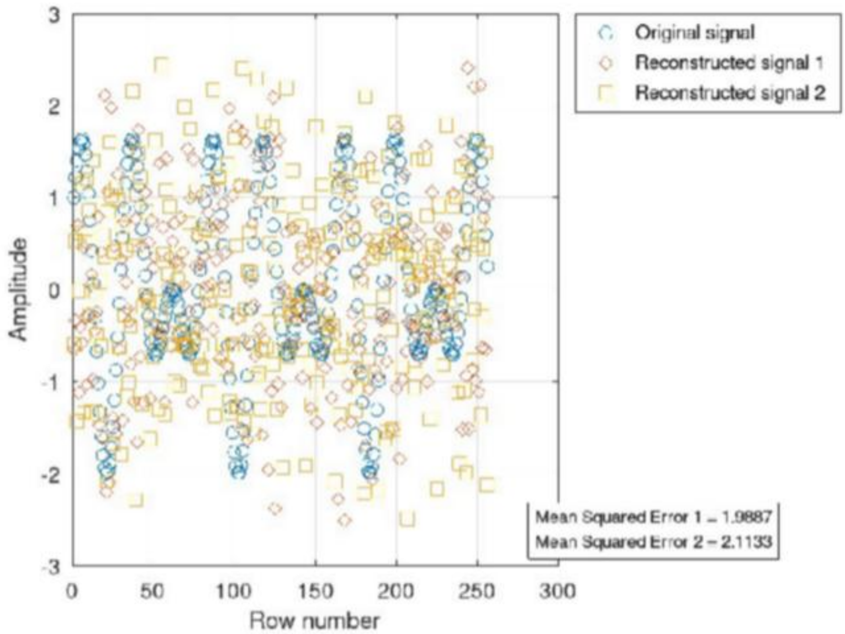

Figure 03. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 10 %

For M = 20 %, Figure 04 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

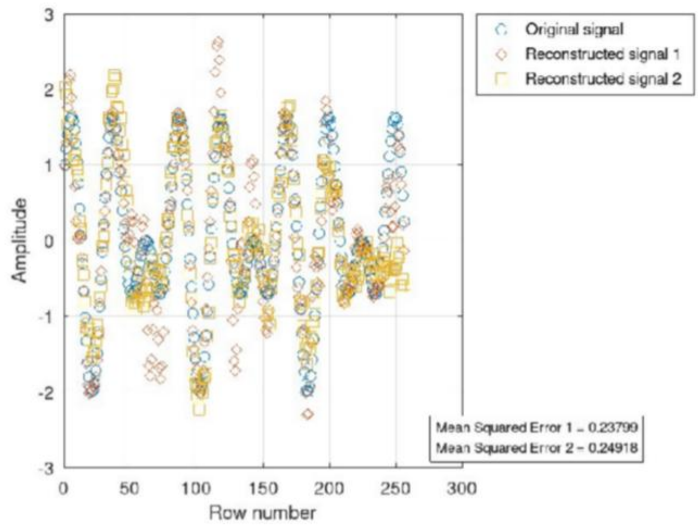

Figure 04. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 20 %

For $M = 30$ %, Figure 05 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

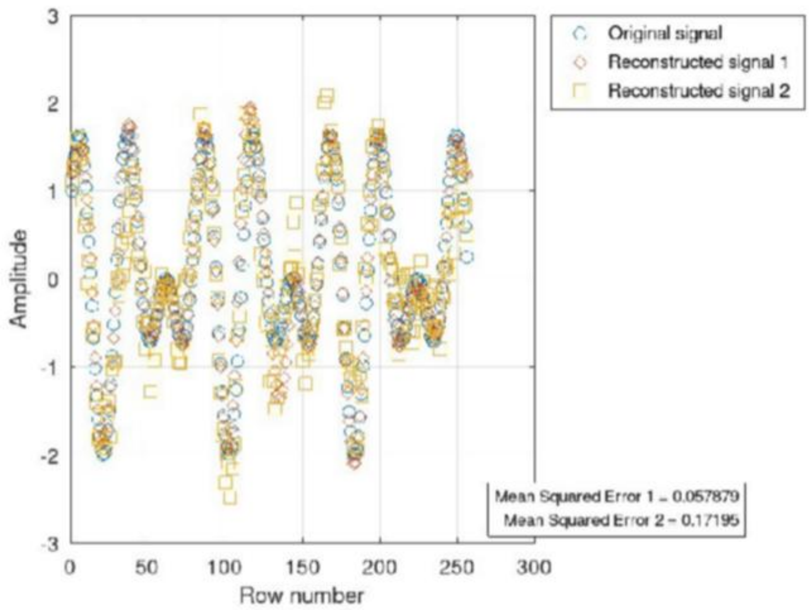

Figure 05. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 30 %

For M = 40 %, Figure 06 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

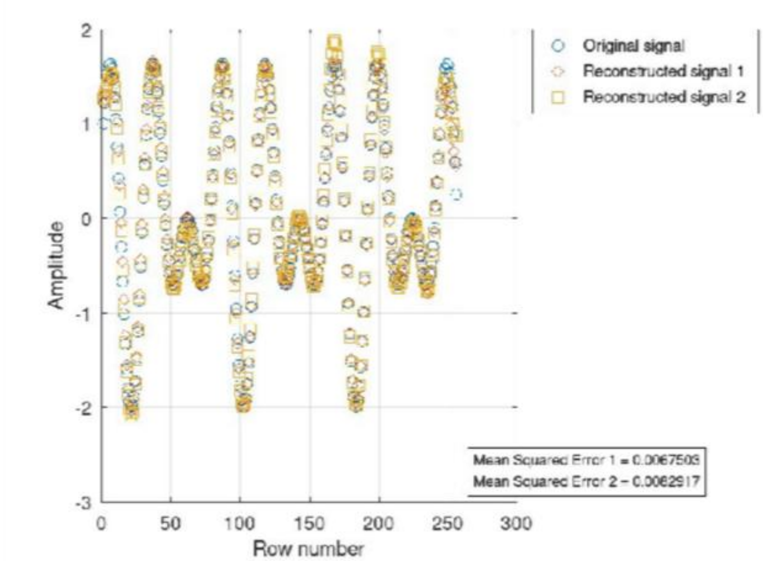

Figure 06. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 40 %

For M = 50 %, Figure 07 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

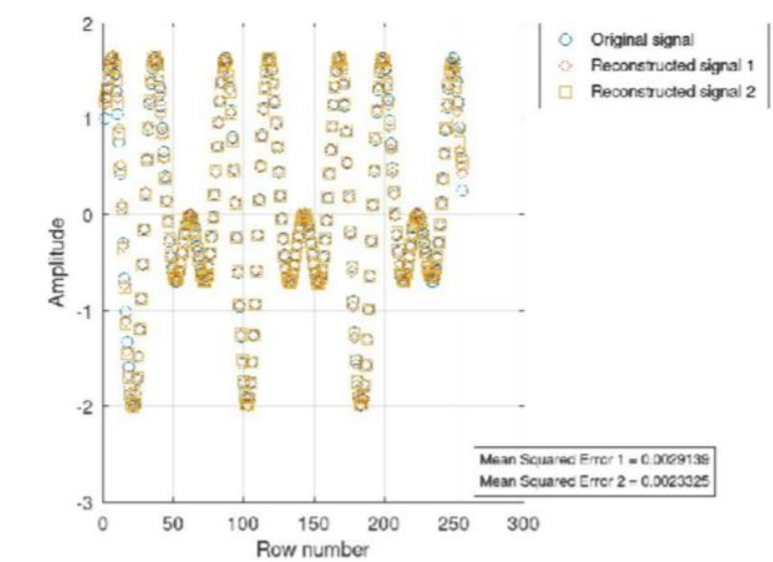

Figure 07. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 50 %

For M = 60 %, Figure 08 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

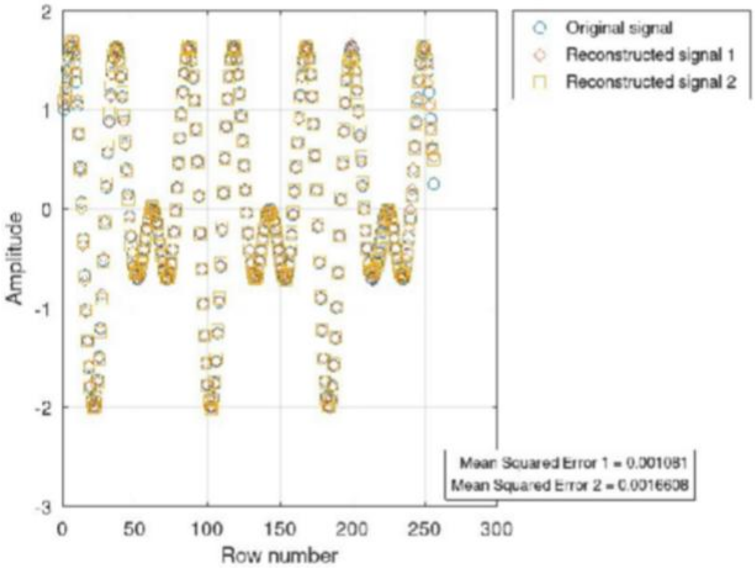

Figure 08. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 60 %

For M = 70 %, Figure 09 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

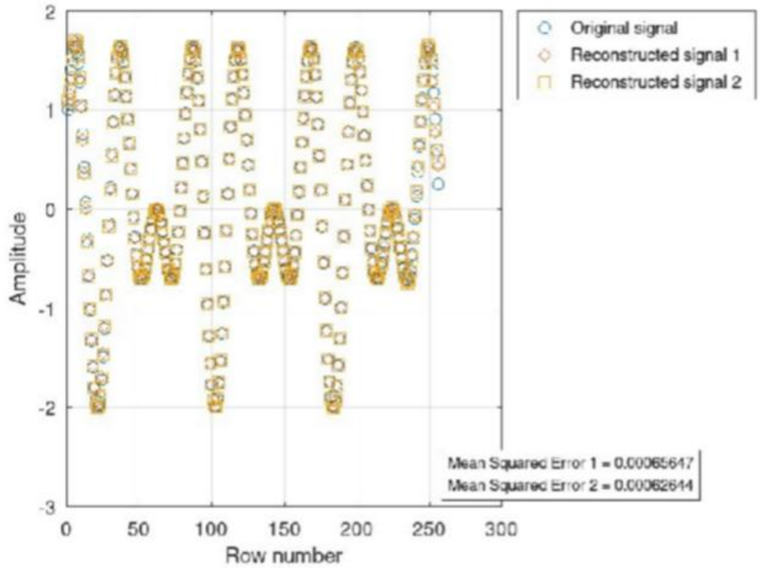

Figure 09. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 70 %

For $M = 80$ %, Figure 10 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

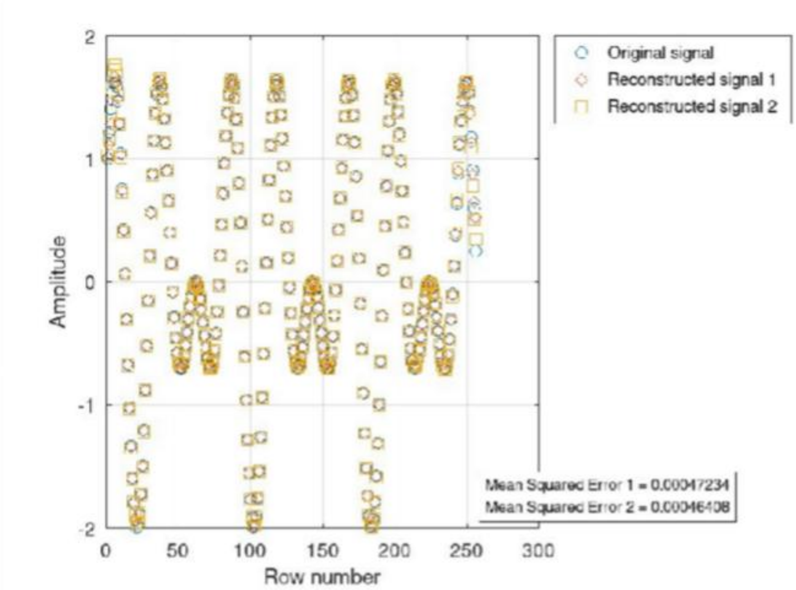

Figure 10. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 80 %

For $M = 90$ %, Figure 11 presents the original signal, the signals reconstructed using approaches 1 and 2 (the latter employing the OMP algorithm), as well as the squared errors between the original signal and each of the reconstructed signals.

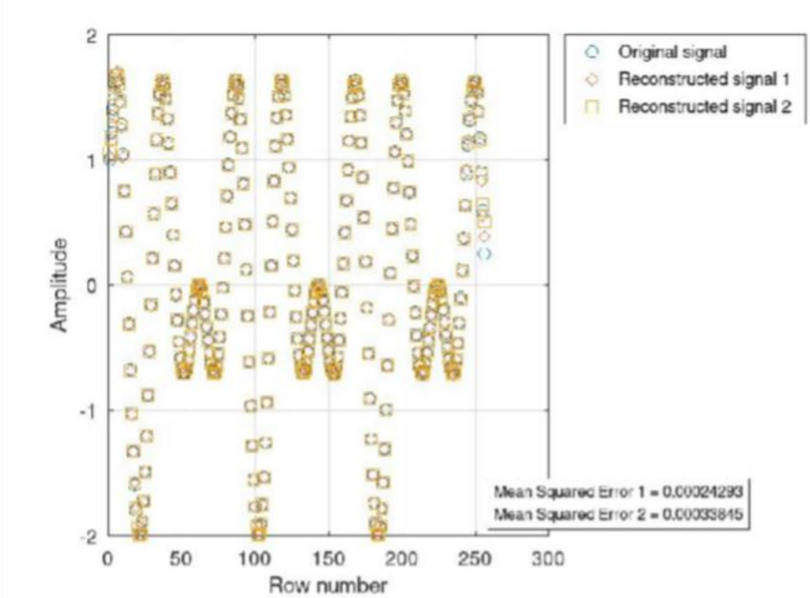

Figure 11. Original signal, reconstructed signals (approach 1, approach 2 with OMP), and corresponding Squared Errors for M = 90 %

By referring to figures 3 to 11, the following table can be constructed :

Table 1. Variation of the mean squared error of each approach with respect to the percentage of M

M	Mean squared error 1	Mean squared error 2
10 %	1.98870	2.11330
20 %	0.23799	0.24918
30 %	0.05787	0.17195
40 %	0.00675	0.00829
50 %	0.00291	0.00233
60 %	0.00108	0.00166
70 %	0.00065	0.00062
80 %	0.00047	0.00046
90 %	0.00024	0.00033

By refering to table 1 presented earlier, the following figure 12 can be introduced :

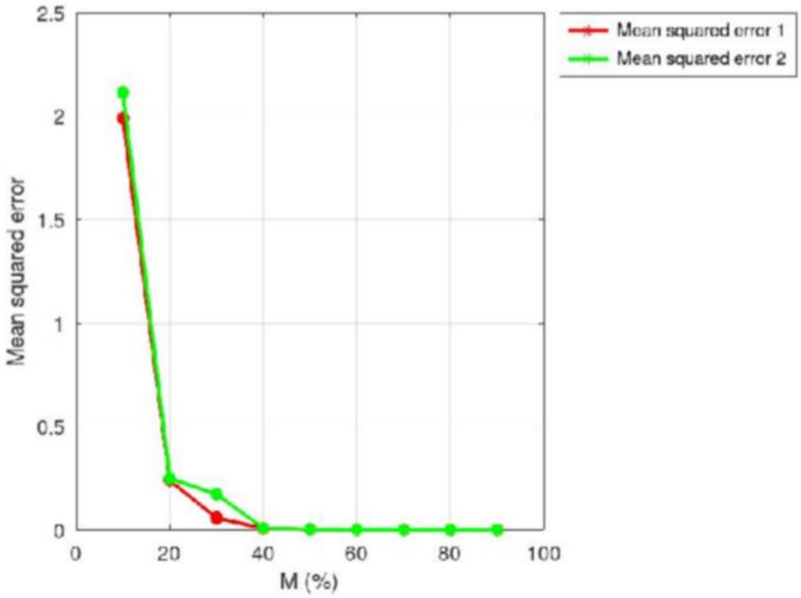

Figure 12. Graphical representation of the mean squared errors for the two approaches

According to Table 1 and Figure 12, it can be observed that both compressive sensing reconstruction approaches exhibit a rapid decrease in mean squared error (MSE) as the measurement percentage M increases, with a particularly sharp drop between 10% and 40 %, highlighting the critical threshold beyond which the signal sparsity can be effectively exploited. However, Approach 1 (MSE1) consistently outperforms Approach 2 (MSE2), especially at low measurement densities (10 % to 30 %), where it shows significantly lower errors, indicating better robustness and accuracy under limited sampling conditions. From 40 % onwards, both methods converge to very low errors, but Approach 1 remains slightly superior, suggesting overall greater efficiency, potentially due to better regularization, a more suitable reconstruction algorithm, or a more favorable measurement matrix, making it the preferred method for applications with a restricted number of measurements.

III. Conclusion

Both approaches demonstrate equivalent reconstruction performance, despite being based on different principles. The similarity of the results suggests that the choice between direct compression and prior transformation should be guided by practical criteria (computational efficiency, ease of implementation) rather than quality gains. To go beyond this equivalence, three avenues for improvement emerge: (1) joint optimization of the matrices φ and ψ through learning, (2) dynamic adaptation of the compression rate M according to the local complexity of the signal, and (3) integration of intelligent post-processing modules to correct residual artifacts. These strategies, applicable to both approaches, could reveal hidden differences between the methods while enhancing their absolute performance—particularly valuable for critical applications.

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